Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Differentiation

Time allowed: 1 hour 40 minutes

Subject code **0580**

Mathematics

Questions from Paper 2 and Paper 4

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
You may use a calculator.

Total marks

/ 80

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 80 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 $y = ax^{11} + 3x^b$

$$\frac{dy}{dx} = 44x^{10} + 18x^c$$

Find the values of a, b and c.

$a =$ .....

$b =$ .....

$c =$ .....

[2]

Differentiation Click for more variations

1	[a =] 4 [b =] 6 [c =] 5	**2**	**B1** for one correct

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2 **(a)** Differentiate $6+4x-x^2$.

..... [2]

(b) Find the coordinates of the turning point of the graph of $y = 6+4x-x^2$.

(..... , ...) [2]

(a) Differentiation (b) Differentiation Click for more variations

2(a)	$4-2x$	**2**	**B1** for 4 or $-2x$
2(b)	(2, 10)	**2**	**B1** for x-coordinate of 2 or **M1** for *their* $4-2x=0$

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3 **(a)** Differentiate $x^3 - 3x^2 + 1$.

..... [2]

(b) Find the coordinates of the turning points of the graph of $y = x^3 - 3x^2 + 1$.

(..... , ...)

(..... , ...)

[4]

(a) Differentiation (b) Differentiation Click for more variations

3(a)	$3x^2 - 6x$ or $3x(x - 2)$	**2**	**B1** for $3x^2 - kx$ or $kx^2 - 6x$ or correct answer spoilt or **SC1** for $3x^2 - 6x + 1$

3(b)	(0, 1) (2, –3)	**4**	**B3** for one correct coordinate or for two correct values of x following $3x^2 - 6x$ oe seen in (b) or **M2** for [3]$x(x - 2)$ [= 0] or $\frac{--6 \pm \sqrt{(-6)^2 - 4 \times 3 \times 0}}{2 \times 3}$ [= 0] oe OR **M1** for *their* $\frac{dy}{dx} = 0$ or stating $\frac{dy}{dx} = 0$ **M1** for correct method to solve *their* quadratic

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4 **(a)** The point $(-1, 6)$ lies on a curve.

This curve has the derived function $\frac{dy}{dx} = -4x^3 - 9x^2 + 5$.

Show that $(-1, 6)$ is a stationary point of the curve.

[2]

(b) A different curve has equation $y = 2x^3 - 6x + 8$.

(i) Calculate the gradient of the tangent to this curve at the point $(-2, 2)$.

..... [3]

(ii) Find the x-coordinates of the stationary points of this curve.

$x =$ ..... and $x =$ ..... [2]

(a) Differentiation (b)(i) Differentiation (b)(ii) Differentiation Click for more variations

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4(a)	$-4(-1)^3 - 9(-1)^2 + 5$ or better	**M1**	
	= 0 [so stationary point]	**A1**	with no errors
4(b)(i)	18	**3**	**B2** for $6x^2 - 6$ isw OR **B1** for $6x^2 + k$ (any k) isw or $px^2 - 6$ isw ($p \neq 0$) or $6x^2 - 6 + 8$ **M1**dep on B1 for $x = -2$ substituted into *their* $\frac{dy}{dx}$
4(b)(ii)	1 and –1	**2**	**M1** for $6x^2 - 6 = 0$ oe seen or for *their* $\frac{dy}{dx} = 0$ if B1 scored in part **(b)(i)**

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5 **(a)** Find the coordinates of the turning points of the graph of $y = x^3 - 12x + 6$.
You must show all your working.

(..... , ...) and (..... , ...) [5]

(b) Determine whether each turning point is a maximum or a minimum.
Show how you decide.

[3]

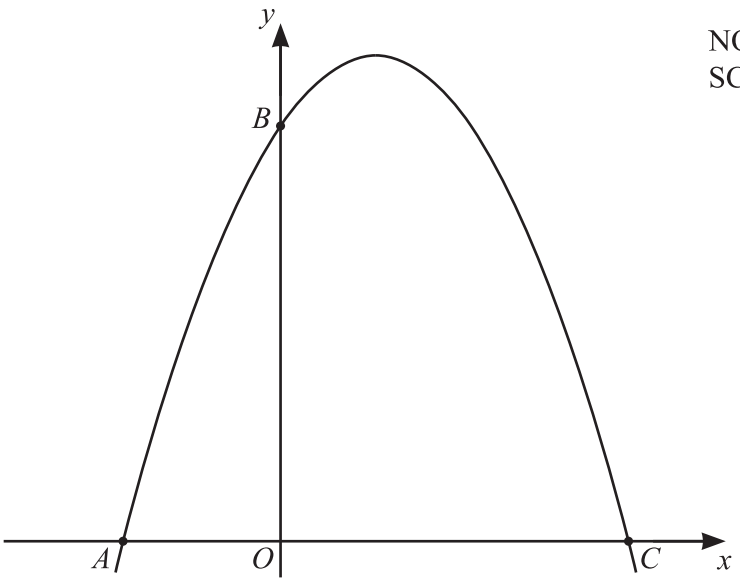
(a) Differentiation (b) Differentiation Click for more variations

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5(a)	(2, –10) and (–2, 22)	**5**	**B2** for $3x^2 - 12$ isw or **B1** for $3x^2 + k$ or $px^2 - 12$ ($p \neq 0$) or for $3x^2 - 12 + 6$ isw **M1** for setting *their* derivative = 0 or $\frac{dy}{dx} = 0$ **B1** for $x = \pm 2$ or for one correct coordinate pair
5(b)	(2, –10) minimum with correct reason or sketch (–2, 22) maximum with correct reason or sketch	**3**	**B2** for 1 correct with correct reasoning or **B2FT** for correct evaluation with correct 2nd derivative for both of *their* different x values or **M1** for showing [2nd derivative =] $6x$ or gradients for one value on either side of one correct stationary point or for reasonable sketch of cubic

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6



NOT TO SCALE

The diagram shows a sketch of $y = 18 + 5x - 2x^2$.

(a) Find the coordinates of the points A, B and C.

A (..... , ...)

B (..... , ...)

C (..... , ...) [4]

(b) Differentiate $18 + 5x - 2x^2$.

..... [2]

(c) Find the coordinates of the point on $y = 18 + 5x - 2x^2$ where the gradient is 17.

(..... , ...) [3]

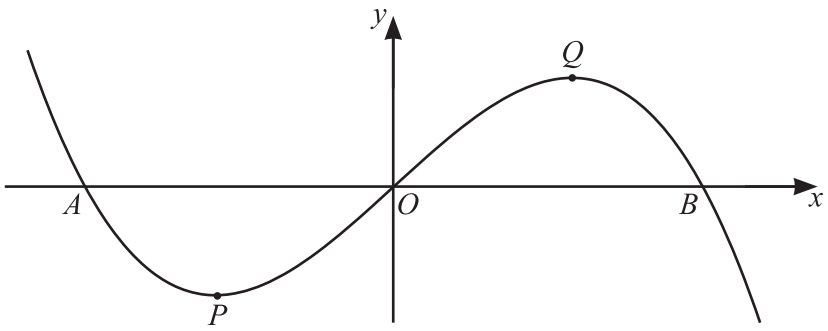
(a) Solving quadratics by factorising (b) Differentiation (c) Differentiation Click for more variations

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6(a)	(–2, 0) (0, 18) (4.5, 0) oe	**4**	**B1** for $B = (0, 18)$ **B3** for $A = (–2, 0)$ and $C = (4.5, 0)$ oe or **B2** for $x = –2$ and $x = 4.5$ oe or **B1** for $(9 – 2x)(2 + x)$ oe or either A or C correct
6(b)	$5 – 4x$ final answer	**2**	**B1** for one correct term when simplified
6(c)	(– 3, –15)	**3**	**B2FT** for $x = – 3$ OR **M1** for *their* **(b)** = 17 **M1 dep** for correct substitution of *their* x into $18 + 5x – 2x^2$ shown

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7



NOT TO SCALE

The diagram shows the graph of $y = 3x - x^3$.
The graph crosses the x-axis at A, at O and at B.
The turning points of the graph are at P and at Q.

(a) Find the x-coordinate of A and the x-coordinate of B.
Give your answers as exact values.

x-coordinate of A .....

x-coordinate of B .....
[3]

(b) **(i)** Differentiate $3x - x^3$.

..... [2]

(ii) Find the coordinates of P and Q.

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P (..... , ...)

Q (..... , ...)
[4]

(a) Solving quadratics by factorising (b)(i) Differentiation (b)(ii) Differentiation Click for more variations

7(a)	[A =] $-\sqrt{3}$ oe [B =] $\sqrt{3}$ oe	**3**	**B2** for $-\sqrt{3}$ oe or $\sqrt{3}$ oe or **M1** for $[x](3-x^2)[=0]$ or better or $\frac{0\pm\sqrt{0^2-4\times-1\times3}}{2\times-1}$ oe
7(b)(i)	$3 - 3x^2$ final answer	**2**	**B1** for 3 or $-3x^2$ correct in an expression or for correct answer spoilt
7(b)(ii)	[P =] (–1, –2) and [Q =] (1, 2)	**4**	**B3** for (–1, –2) or (1, 2) or for two correct values of x or **M2** for $x^2 = 1$ or for [3](1 – x)(1 + x) [= 0] oe factorised or $\frac{0\pm\sqrt{0^2-4\times-3\times3}}{2\times-3}$ oe OR **M1** for [3](1 – x^2) [= 0] or *their* **(b)(i)** = 0 or for stating $\frac{dy}{dx} = 0$ **M1** for correct method to solve *their* quadratic

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8 **(a)** $y = x^4 - 4x^3$

(i) Find the value of y when $x = -1$.

$y =$ ..... [2]

(ii) Find the two stationary points on the graph of $y = x^4 - 4x^3$.

(..... , ...)

(..... , ...) [6]

(b) $y = x^p + 2x^q$

$\frac{dy}{dx} = 11x^{10} + 10x^4$, where $\frac{dy}{dx}$ is the derived function.

Find the value of p and the value of q.

$p =$ .....

$q =$ ..... [2]

(a)(i) Substitution (a)(ii) Differentiation (b) Differentiation Click for more variations

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8(a)(i)	5	**2**	**M1** for $(-1)^4 - 4(-1)^3$
8(a)(ii)	(0, 0) and (3, –27)	**6**	**B2** for $4x^3 - 12x^2$ [= 0] or **B1** for $4x^3$ or $12x^2$ AND **M1** for derivative = 0 or *their* derivative = 0 **M1** for $4x^2(x - 3)$[= 0] **B1** for [x =] 0 and [x =] 3 or [y =] 0 and [y =] –27 or for one correct coordinate pair
8(b)	[p =] 11 [q =] 5	**2**	**B1** for each or **M1** for $\frac{dy}{dx} = px^{p-1} + 2qx^{q-1}$

9 $y = x^7 - 7x^6$

(a) Find the derivative of y with respect to x.

..... [2]

(b) Find the equation of the tangent to the graph of $y = x^7 - 7x^6$ at the point where $x = -1$.
Give your answer in the form $y = mx + c$.

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$y =$ ..... [4]

(c) The graph of $y = x^7 - 7x^6$ has two turning points.

Find the coordinates of these points.
You must show all your working.

(..... , ...)

(..... , ...) [5]

(a) Differentiation (b) Differentiation (c) Differentiation Click for more variations

9(a)	$7x^6 - 42x^5$ final answer	**2**	**B1** for one correct term $7x^6$ or $42x^5$ or for $7x^6 - 42x^5$ seen and spoiled
9(b)	$49x + 41$	**4**	**M1** for substituting $x = -1$ into $[y =] x^7 - 7x^6$ **M1** for $x = -1$ substituted in *their* (a) or the correct derivative to give *their m* **M1** for *their* $-8 =$ (*their m*)$(-1) + c$ oe
9(c)	(0, 0) (6, –46 656)	**5**	**B4** for (6, –46 656) or **B3** for $x = 0$ and 6 **OR** **M1** for *their* $\frac{dy}{dx} = 0$ or stating $\frac{dy}{dx} = 0$ and **M1** for a correct method to solve *their* $7x^6 - 42x^5$

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10 $f(x) = x(x-1)(x-2)$

(a) Find the coordinates of the points where the graph of $y = f(x)$ crosses the x-axis.

(..... , ...)

(..... , ...)

(..... , ...) [2]

(b) Show that $f(x) = x^3 - 3x^2 + 2x$.

[2]

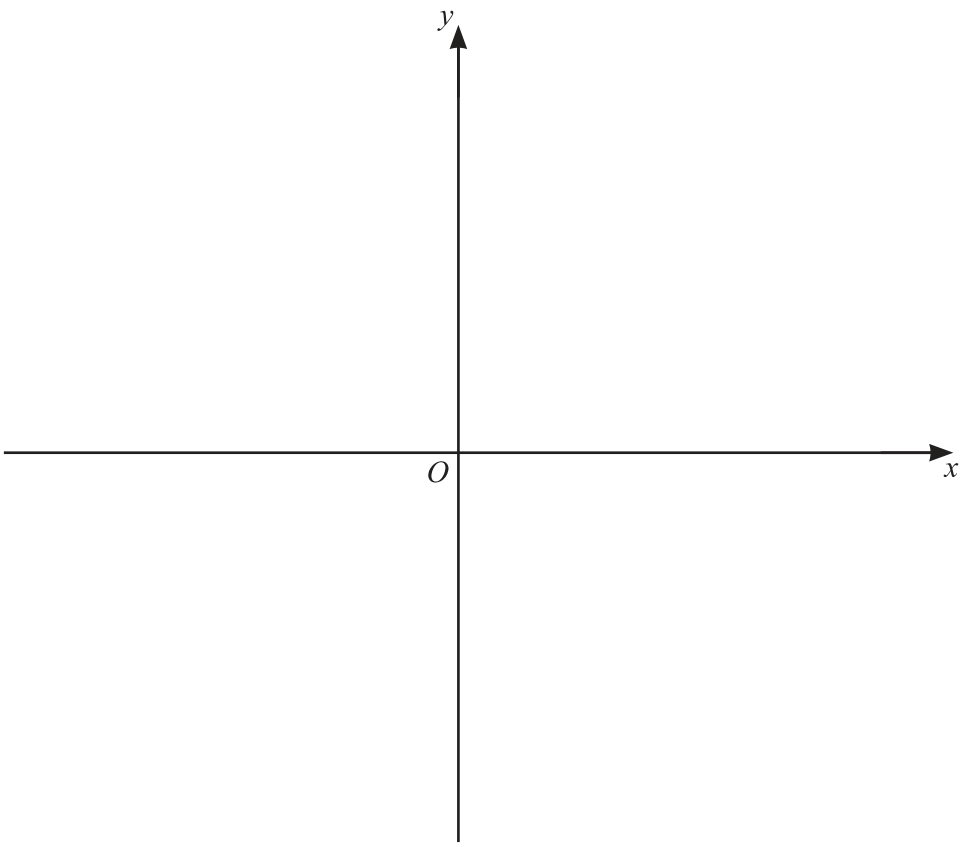
(c) Find the coordinates of the turning points of the graph of $y = f(x)$.
Show all your working and give your answers correct to 1 decimal place.

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(..... , ...)

(..... , ...) [8]

(d) Sketch the graph of $y = \mathrm{f}(x)$.



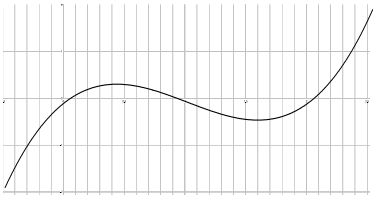
[2]

(a) Plotting curves and graphical solutions (b) Expanding triple brackets (c) Differentiation (d) Differentiation

Click for more variations

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10(a)	(0, 0), (1, 0), (2, 0)	**2**	**B1** for any two correct If 0 scored, SC1 for all three x values clearly identified

10(b)	$x\left(x^2-x-2x+2\right)$ or $\left(x^2-x\right)(x-2)$ or $(x-1)\left(x^2-2x\right)$ leading to x^3-3x^2+2x with no errors or omissions	**2**	**B1** for $x\left(x^2-x-2x+2\right)$ or $\left(x^2-x\right)(x-2)$ or $(x-1)\left(x^2-2x\right)$
10(c)	$3x^2-6x+2$	**B2**	**B1** for 2 correct terms
	their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$	**M1**	
	their $\frac{-(-6)\pm\sqrt{(-6)^2-4(3)(2)}}{2(3)}$	**M2**	**M1** for $\sqrt{(-6)^2-4(3)(2)}$ or for $p=-(-6)$ and $r=2(3)$ if in form $\frac{p\pm\sqrt{q}}{r}$
	(0.4, 0.4) (1.6, –0.4)	**B3**	**B2** for 0.4 or 0.42... **and** 1.6 or 1.57 to 1.58 or for one correct pair of coordinates or **B1** for 0.4 or 0.42... **or** 1.6 or 1.57 to 1.58 If 0 scored **SC1** for $1+\sqrt{\frac{1}{3}}$ and $1-\sqrt{\frac{1}{3}}$ or better or for one correct pair of coordinates in any form
10(d)	Correct sketch 	**2**	**FT** *their* **(c)** but must be cubic i.e. correct shape cubic through origin and max and min in correct quadrants **B1** for cubic shape sketch

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