Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Area under a curve

Time allowed: 1 hour 30 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 2-5, 10 (Paper 1): no calculator.
Questions 1, 6-9 (Paper 2): calculator allowed.

Total marks

/ 57

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 57 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 $y = \dfrac{\sec^2 5x - \tan^2 5x}{\operatorname{cosec} 5x}$

Show that $y = a\sin bx$, where a and b are integers, and hence find the value of $\int_0^{\frac{\pi}{5}} y\,\mathrm{d}x$. [4]

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1	$[y =] \dfrac{1}{\operatorname{cosec} 5x} = \sin 5x$ nfww	**B1**	
	$\int_0^{\frac{\pi}{5}} y\,\mathrm{d}x = \left[-\dfrac{\cos 5x}{5}\right]_0^{\frac{\pi}{5}}$	**B1**	**FT** *their* $a\sin 5x$
	$-\dfrac{1}{5}\cos\left(5\times\dfrac{\pi}{5}\right) - \left(-\dfrac{1}{5}\cos(5\times 0)\right)$	**M1**	**FT** *their* $a(k\cos bx)$ where $k < 0$ or $k = \dfrac{1}{5}$
	$\dfrac{2}{5}$	**A1**	

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2 Find the exact value of $\int_0^{\frac{\pi}{2}} (\cos 3x + 4\sin 2x + 1)\,\mathrm{d}x$. [5]

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2	$\frac{1}{3}\sin 3x - 2\cos 2x + x$	**2**	**M1** for $a\sin 3x + b\cos 2x + x$, $a \neq \pm 3$ and $b \neq \pm 8$ **A1** all correct
	$\left(\frac{1}{3}\sin\frac{3\pi}{2} - 2\cos\pi + \frac{\pi}{2}\right) - (-2)$	**M1**	**Dep** on previous **M** mark for correct substitution (seen or implied) of **both** limits in x
	$\frac{11}{3}$	**A1**	
	$\frac{\pi}{2}$	**B1**	From correct substitution (seen or implied) of **both** limits in x

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3 Find $\int_0^2 \left(1+e^{2x}\right)^2 dx$, giving your answer in exact form. [5]

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3	$1+2\mathrm{e}^{2x}+\mathrm{e}^{4x}$	**B1**	For correct integrand
	$x+\mathrm{e}^{2x}+\dfrac{\mathrm{e}^{4x}}{4}$	**B2**	**B1** for 2 correct terms in integral
	$\left(2+\mathrm{e}^{4}+\dfrac{\mathrm{e}^{8}}{4}\right)-\left(1+\dfrac{1}{4}\right)$	**M1**	For correct application of limits. Must be in the form $m\mathrm{e}^{4x}+n\mathrm{e}^{2x}+x$
	$\dfrac{3}{4}+\mathrm{e}^{4}+\dfrac{\mathrm{e}^{8}}{4}$	**A1**	

4 Show that $\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\left((\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2\right)\mathrm{d}\theta=k\pi$, where k is an integer to be found. [5]

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4	$(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2=2$	**2**	**M1** for expansion of both terms with an attempt to simplify either using the correct identity or eliminating terms in $2\sin\theta\cos\theta$
	$\displaystyle\int 2\,\mathrm{d}\theta=2\theta$	**M1**	**Dep M1**
	$\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}2\,\mathrm{d}\theta=3\pi-\pi$ $=2\pi$	**2**	**Dep M1** for correct use of limits

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5 Given that $\int_0^{2a+1} \frac{8}{4x+3}\,\mathrm{d}x = \ln 16,$ find the exact value of the constant a. [5]

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5	$\ln(4x+3)$	**B1**	
	$2\ln(8a+7)-2\ln(3)\ \ (=\ln 16)$	**M1**	**Dep** for correct application of limits in *their* $k\ln(4x+3)$
	$(2)\ln\frac{8a+7}{3}$ oe	**M1**	**Dep** for use of division rule
	$\ln 16 = 2\ln 4$ oe	**B1**	
	$a=\frac{5}{8}$ only	**A1**	

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6 **(i)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{5}{3x+2}\right)$. [2]

(ii) Use your answer to part **(i)** to find $\int \frac{30}{(3x+2)^2}\,\mathrm{d}x$. [2]

(iii) Hence evaluate $\int_1^2 \frac{30}{(3x+2)^2}\,\mathrm{d}x$. [2]

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Differentiation, tangents, normals and stationary points Click for more variations

6(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{5}{3x+2}\right)=-5(3x+2)^{-2}\times 3$	**M1**	$-5(3x+2)^{-2}$
		A1	$\times 3$
6(ii)	$\int \frac{30}{(3x+2)^2}\,\mathrm{d}x=\left[\frac{-10}{(3x+2)}\right]$	**M1**	$\frac{1}{(3x+2)}$
		A1	$\times -10$
6(iii)	$\left[\frac{-10}{(3x+2)}\right]_1^2=-\frac{10}{8}+\frac{10}{5}$	**M1**	Insert limits and subtract
	$=\frac{3}{4}$	**A1**	

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7 Find the exact value of $\int_2^4 \frac{(x+1)^2}{x^2} \mathrm{d}x$. [6]

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7	$\frac{(x+1)^2}{x^2}=\frac{x^2+2x+1}{x^2}=1+\frac{2}{x}+\frac{1}{x^2}$	**2**	**B1** for expanding numerator seen anywhere. **M1** for attempt to divide *their* three term numerator by x^2.
	$\int 1+\frac{2}{x}+\frac{1}{x^2}\mathrm{d}x=x+2\ln x-\frac{1}{x}+(c)$	**2**	**A2/1/0** minus 1 each error or omission.
	$\left[4-2\ln 4-\frac{1}{4}\right]-\left[2+2\ln 2-\frac{1}{2}\right]$	**M1**	**Dep** insert 4 and 2 into *their* three or two term integrand and subtract correctly.
	$=\frac{9}{4}+2\ln 2$	**A1**	oe must be exact two terms. isw

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8 **(a)** Find $\int\left(4x+5-\frac{1}{2x+3}\right)\mathrm{d}x$. [3]

(b) Hence find the exact value of $\int_1^3\left(4x+5-\frac{1}{2x+3}\right)\mathrm{d}x$, simplifying your answer. [3]

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8(a)	$2x^2+5x-\frac{1}{2}\ln(2x+3)+c$ oe	**B3**	**B2** for $2x^2+5x-\frac{1}{2}\ln(2x+3)$ or $2x^2+5x-\frac{1}{2}\ln 2x+3+c$ or $2x^2+5x+k\ln(2x+3)+c$ with $k\neq 0$ or **B1** for $2x^2+5x+\ldots+c$ or $\ldots-\frac{1}{2}\ln 2x+3$ or $\ldots+k\ln(2x+3)$ with $k\neq 0$
8(b)	Substitutes limits and subtracts in correct order	**M1**	**FT** *their* part (a) providing it includes a term $k\ln(2x+3)$ with $k\neq 0$
	$\left[18+15-\frac{1}{2}\ln 9\right]-\left[2+5-\frac{1}{2}\ln 5\right]$	**A1**	
	$26-\frac{1}{2}\ln\frac{9}{5}$ or $26+\frac{1}{2}\ln\frac{5}{9}$ oe	**A1**	

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9 A particle travels in a straight line so that, t seconds after passing a fixed point, its velocity, $v\,\text{ms}^{-1}$, is given by

$$v = \mathrm{e}^{\frac{t}{4}} \quad \text{for } 0 \leqslant t \leqslant 4,$$

$$v = \frac{16\mathrm{e}}{t^2} \quad \text{for } 4 \leqslant t \leqslant k.$$

The total distance travelled by the particle between $t = 0$ and $t = k$ is 13.4 metres. Find the value of k. [6]

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9	$\int e^{\frac{t}{4}}\,dt = 4e^{\frac{t}{4}}(+c)$ and $\int \frac{16e}{t^2}dt = \frac{-16e}{t}(+c)$	**B3**	**B2** for either correct or **B1** for $\int e^{\frac{t}{4}}\,dt = ae^{\frac{t}{4}}(+c)$ where a is a constant, $a > 0$ or $\int \frac{16e}{t^2}dt = \frac{-b}{t}(+c)$ where b is a constant $b > 0$
	Correct plan: $\int_0^4 e^{\frac{t}{4}}\,dt + \int_4^k \frac{16e}{t^2}dt = 13.4$ soi	**M1**	
	Correct equation: $4e^1 - 4e^0 + \left(-\frac{16e}{k} + \frac{16e}{4}\right) = 13.4$ OR [When $t = 4$ $s = 4e - 4$ When $t = k$ $s = \frac{-16e}{k} + 8e - 4$ and] $13.4 = \frac{-16e}{k} + 8e - 4$	**A1**	**dep** on **B3**; implies **M1**
	$k = 10$ or awrt 10.0	**A1**	dep on all previous marks awarded

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10 **(a)** Find $\int_2^4 (5x-2)^{-\frac{2}{3}}\,\mathrm{d}x$, giving your answer in exact form. [4]

(b) Find $\int_0^{\frac{1}{2}}\left(\frac{4}{2x+1}+\frac{8}{(2x+1)^2}\right)\mathrm{d}x$, giving your answer in the form $a+\ln b$, where a and b are integers. [5]

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10(a)	$a(5x-2)^{\frac{1}{3}}$	**M1**	
	$\frac{3}{5}(5x-2)^{\frac{1}{3}}$ oe	**A1**	
	$\frac{3}{5}\left(18^{\frac{1}{3}}-2\right)$ or exact equivalent	**2**	**Dependent M1** for correct use of limits

10(b)	$2\ln(2x+1)$ oe	**B1**	
	$-\frac{4}{2x+1}$ oe	**B1**	
	$\left(2\ln 2-\frac{4}{2}\right)-(-4)$	**M1**	For correct substitution of limits, must be using the form $a\ln(2x+1)+\frac{b}{2x+1}$
	$\ln 4+2$	**2**	**A1** for each term

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