Mark scheme

Arithmetic and geometric progressions

Additional Mathematics 0606 · Questions from Paper 1 and Paper 2

Total marks

88

MARKS FOR EACH QUESTION

1	2	3	4	5	6	7	8	9	10	Total
6	6	7	7	7	7	10	11	12	15	**88**

Question 1 [6]

1(a)	$a+6d=158$ $a+9d=149$	**B1**	For both equations, may be implied by a correct a and d
	$d=-3,$	**B1**	
	$a=176$	**B1**	

1(b)	$\frac{n}{2}\left(352+(n-1)(-3)\right) \quad (<0)$	**M1**	For correct attempt at sum formula with *their a* and *their d*,
	$\frac{355}{3}$ or 118.3 oe	**A1**	
	119	**A1**	

Question 2 [6]

2(a)	$a+ar=9$ or $\frac{a\left(1-r^2\right)}{1-r}=9$ or $\frac{a\left(r^2-1\right)}{r-1}=9$	**B1**	
	$\frac{a}{1-r}=25$	**B1**	
	$r^2=\frac{16}{25}$	**B1**	**Dep B** mark on both previous **B** marks
	$r=\pm\frac{4}{5}$ or ± 0.8	**B1**	**Dep B1** mark on all previous **B** marks
2(b)	$a=5,\ 45$	**2**	**B1** for each

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Question 3 **[7]**

3(a)	$r^3 = \frac{1}{8}$ soi	**M1**	Allow unsimplified $ar^{14} = \frac{1}{8}ar^{11}$ or $\frac{5r^{11} - 5r^{12}}{5r^{14} - 5r^{15}} = 8$ oe
	$r = \frac{1}{2}$	**A1**	
	$5 = \frac{a}{1-r}$	**M1**	For use of sum to infinity with *their* r, must be $-1 < r < 1$
	$a = \frac{5}{2}$	**A1**	
3(b)	$their(\mathbf{a}) \times \frac{\left(1-(their\ r)^n\right)}{(1-their\ r)}$	**M1**	For use of the sum to n terms
	$(their\ r)^n = 0.0002$ (12.29)	**M1**	**M1 dep** For simplification and attempt to obtain the critical value using either an equation or an inequality leading to $n =$ or $n >$
	13	**A1**	Accept $n \geqslant 13$

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Question 4 **[7]**

4(a)	$d = 3\log_x 3$ or $\log_x 3^3$ nfww	**B1**	Must be exact Allow $d = \log_x 27$ nfww
	$\frac{n}{2}\left(2\log_x 3 + 3(n-1)\log_x 3\right)$ nfww	**M1**	For use of sum formula with *their d* must be in the form of $\log_x 3$
	$\frac{n}{2}(3n-1)\log_x 3$ or $\left(\frac{3n^2}{2} - \frac{n}{2}\right)\log_x 3$	**A1**	Must be in the form of $k\log_x 3$
4(b)	$r = 3\tan^2\theta$	**B1**	soi
	$\lvert 3\tan^2\theta\rvert < 1$ or $[-1 <]3\tan^2\theta < 1$ or $\left[-\frac{1}{3} <\right]\tan^2\theta < \frac{1}{3}$ or $[0 <]\tan^2\theta < \frac{1}{3}$	**B1**	
	$\tan\theta < \frac{1}{\sqrt{3}}$ or $0 < \tan\theta < \frac{1}{\sqrt{3}}$	**B1**	
	$0 < \theta < \frac{\pi}{6}$	**B1**	

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Question 5 **[7]**

5(a)	$10r = 10 + 3d$ soi	**B1**	
	$10r^2 = 10 + 5d$ soi	**B1**	
	$10\left(\frac{10+3d}{10}\right)^2 = 10 + 5d$ oe or $10r^2 = 10 + \frac{5(10r-10)}{3}$ oe	**M1**	
	$9d^2 + 10d = 0$ or $3r^2 - 5r + 2 = 0$ oe	**A1**	
	Correct method to find d or r	**A1**	**FT** *their* quadratic in d or r
	$d = -\frac{10}{9}$ and $r = \frac{2}{3}$	**A1**	
5(b)	Appropriate determination for *their r* e.g. $\lvert r \rvert < 1$ so the geometric progression has a sum to infinity oe	**B1**	**STRICT FT** *their r*

Question 6 **[7]**

6(a)	$ar^3 = \frac{8k^6}{27}$, $ar^5 = \frac{32k^{10}}{243}$ soi	**B1**	May be implied by a correct 5th term $\frac{16k^8}{81}$
	$r = \frac{2k^2}{3}$	**2**	**M1** for solution of *their* equations to obtain either $r = mk^2$ or $a = c$, where m is an unsimplified numeric constant and c is a non-zero constant e.g. Use of $r^2 = \frac{\text{6th term}}{\text{4th term}}$
	$a = 1$	**A1**	
6(b)	$\frac{1}{1 - \frac{2k^2}{3}} = 3$ soi	**B1**	For use of sum to infinity formula with *their* values of a where a is numeric and r which is in terms of k^2
	$k = \pm 1$	**2**	**B1** for each must be from correct work

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Question 7 **[10]**

7(a)	Correct pair of simplified linear equations in a and d with terms collected, e.g., $3a+3d=-36$ isw or $a+d=-12$ isw $3a+30d=72$ isw or $a+10d=24$ isw	**B3**	**B2** for one correct simplified equation or **B1** for $a+a+d+a+2d=-36$ or $\frac{3}{2}\{2a+(3-1)d\}=-36$ or $a+9d+a+10d+a+11d=72$ or $\frac{12}{2}\{2a+(12-1)d\}-\frac{9}{2}\{2a+(9-1)d\}=72$ or $12a+66d-9a-36d=72$ or $\frac{3}{2}\{2(a+9d)+(3-1)d\}=72$
	Solves two linear equations for d or a e.g. $27d=108 \rightarrow d=\ldots$ or $9d=36 \rightarrow d=\ldots$ or $a+10(-12-a)=24 \rightarrow a=\ldots$ $27a=-432 \rightarrow a=\ldots$	**M1**	**FT** *their* linear equations in a and d providing at least **B1** earned and the equations have a solution
	$d=4$ and $a=-16$ nfww	**A1**	
7(b)	1.2^n *101	**B3**	where * is any inequality sign or =; **B2** for $\frac{[1](1.2^n-1)}{(1.2-1)}*500$ or **B1** for $r=1.2$ soi
	$n\log 1.2*\log 101$ or $\log_{1.2}101$ soi	**M1**	**FT** 1.2^n * *their* 101 providing **B2** has been awarded and (*their* 101) > 0
	$n=26$	**A1**	**dep** on all previous marks awarded

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Question 8 **[11]**

8(a)(i)	$\frac{1}{8}$ or 0.125	**2**	**M1** for $64(0.5)^9$ oe
8(a)(ii)	$\frac{1023}{8}$ or 127.875	**2**	**M1** for $\frac{64(1-0.5^{10})}{1-0.5}$ oe
8(a)(iii)	128	**1**	
8(b)	$\frac{20}{2}\{2a+19d\}-400=2\times\frac{10}{2}\{2a+9d\}$ oe, soi	**M2**	**M1** for $\frac{20}{2}\{2a+19d\}$ or $\frac{10}{2}\{2a+9d\}$ soi
	$5a = a + 5d$ soi	**M1**	
	$d = 4$	**A1**	
	$a = 5$	**A1**	
	27 nfww	**B1**	must have earned all previous marks

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Question 9 **[12]**

9(a)	Attempts to solve $a + 2d = 13$ and $a + 9d = 41$ oe	**M2**	**M1** for $a + 2d = 13$ and $a + 9d = 41$ soi
	$d = 4$ and $a = 5$	**A2**	**A1** for $d = 4$ or $a = 5$
9(b)	$\frac{n}{2}\{2(5)+(n-1)4\}$ soi	**M1**	**FT** *their a* and *their d*
	$2n^2 + 3n - 2555\,[*0]$	**A1**	where * could be = or any inequality sign
	Solves *their* 3-term quadratic of the form $ax^2 + bx + c$ [*0] by factorising or formula or *their* 3-term quadratic of the form $ax^2 + bx * c$ or better if completing the square	**M1**	
	35	**A1**	

9(c)	May work ***consistently*** in n throughout but must conclude in k to earn the final mark		
	$S_{2k} = \frac{2k}{2}\{10+(2k-1)4\}$ soi	**B1**	**FT** *their a* and *their d*
	$\frac{2k}{2}\{10+(2k-1)4\} - \frac{k}{2}\{10+(k-1)4\}$ soi	**M1**	**FT** *their a* and *their d* ; condone at most one error
	Simplifies as far as e.g. $8k^2 + 6k - (3k + 2k^2)$ or $8k^2 + 6k - 3k - 2k^2$	**A1**	
	Correct completion to given answer: $6k^2 + 3k = 3k(1 + 2k)$	**A1**	
	Alternative method		
	$\frac{2k}{2}\{2a+(2k-1)d\}$ **and** a = *their* 5 and d = *their* 4 substituted at some point	**(B1)**	
	$ak - \frac{d}{2}k + \frac{3}{2}dk^2$ oe	**(M1)**	condone at most one error
	$5k - \frac{4}{2}k + \frac{3}{2}\times 4\times k^2$	**(A1)**	
	Correct completion to given answer: $6k^2 + 3k = 3k(1 + 2k)$	**(A1)**	

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Question 10 **[15]**

10(a)(i)	$a + d$, $a + 13d$, $a + 16d$ soi	**B1**	
	$\frac{a+13d}{a+d} = \frac{a+16d}{a+13d}$ oe	**M2**	**FT** *their* 3 distinct terms providing of the form $a + kd$ where $k \neq 0$ and at least one is correct **M1** for either $[r =]\frac{a+13d}{a+d}$ or $[r =]\frac{a+16d}{a+13d}$ or $[r =]\sqrt{\frac{a+16d}{a+d}}$
	Clears fractions and expands oe: $a^2 + 26ad + 169d^2$ $= a^2 + 17ad + 16d^2$	**A1**	
	$9ad + 153d^2 = 0$ or $9ad = -153d^2$	**A1**	
	Convincingly derives $a = -17d$ e.g. $9d(a + 17d) = 0$ [therefore] $a = -17d$ oe	**A1**	

10(a)(ii)	$r = 0.25$ oe	**2**	**M1** $\frac{-17d+13d}{-17d+d}$ or $\frac{-17d+16d}{-17d+13d}$ or $-16d$, $-4d$, $-d$
10(b)	$\frac{q}{1-0.25} = \frac{256}{3}$ oe	**M1**	**FT** *their* 0.25 providing it is between –1 and 1
	$q = 64$	**A1**	
	$[a + d =$ *their* 64 $]$ $-17d + d =$ *their* 64 or $a - \frac{a}{17} =$ *their* 64	**DM1**	**dep** on previous **M1**
	$d = -4$ and $a = 68$ oe OR $d = -4$ and $S_{20} = -150d$ oe	**A2**	**A1** for either correct
	$S_{20} = \frac{20}{2}\{2(\text{their}68) + 19(\text{their}(-4))\}$ or $S_{20} = -150(\text{their } d)$	**M1**	**FT** *their* a and d
	600	**A1**	

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