Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Arithmetic and geometric progressions

Time allowed: 2 hours 15 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 1-6 (Paper 1): no calculator.
Questions 7-10 (Paper 2): calculator allowed.

Total marks

/ 88

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 88 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 The 7th and 10th terms of an arithmetic progression are 158 and 149 respectively.

(a) Find the common difference and the first term of the progression. [3]

(b) Find the least number of terms of the progression for their sum to be negative. [3]

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1(a)	$a+6d=158$ $a+9d=149$	**B1**	For both equations, may be implied by a correct a and d
	$d=-3,$	**B1**	
	$a=176$	**B1**	

1(b)	$\frac{n}{2}\left(352+\left(n-1\right)\left(-3\right)\right)\qquad\left(<0\right)$	**M1**	For correct attempt at sum formula with *their a* and *their d*,
	$\frac{355}{3}$ or 118.3 oe	**A1**	
	119	**A1**	

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2 The sum of the first two terms of a geometric progression is 9.
The sum to infinity of this geometric progression is 25.

(a) Find the possible values of the common ratio of this geometric progression. [4]

(b) Find the first term of this geometric progression for each possible value of the common ratio. [2]

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2(a)	$a+ar=9$ or $\frac{a\left(1-r^2\right)}{1-r}=9$ or $\frac{a\left(r^2-1\right)}{r-1}=9$	**B1**	
	$\frac{a}{1-r}=25$	**B1**	
	$r^2=\frac{16}{25}$	**B1**	**Dep B** mark on both previous **B** marks
	$r=\pm\frac{4}{5}$ or ± 0.8	**B1**	**Dep B1** mark on all previous **B** marks
2(b)	$a=5,\ 45$	**2**	**B1** for each

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3 A geometric progression is such that the fifteenth term is equal to $\frac{1}{8}$ of the twelfth term. The sum to infinity is 5.

(a) Find the first term and the common ratio. [4]

(b) Find the least number of terms needed for the sum of the geometric progression to be greater than 4.999. [3]

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3(a)	$r^3 = \frac{1}{8}$ soi	**M1**	Allow unsimplified $ar^{14} = \frac{1}{8}ar^{11}$ or $\frac{5r^{11} - 5r^{12}}{5r^{14} - 5r^{15}} = 8$ oe
	$r = \frac{1}{2}$	**A1**	
	$5 = \frac{a}{1-r}$	**M1**	For use of sum to infinity with *their* r, must be $-1 < r < 1$
	$a = \frac{5}{2}$	**A1**	
3(b)	$their(\mathbf{a}) \times \frac{\left(1-(their\ r)^n\right)}{(1-their\ r)}$	**M1**	For use of the sum to n terms
	$(their\ r)^n = 0.0002$ (12.29)	**M1**	**M1 dep** For simplification and attempt to obtain the critical value using either an equation or an inequality leading to $n =$ or $n >$
	13	**A1**	Accept $n \geqslant 13$

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4 **(a)** The first 3 terms of an arithmetic progression are $\log_x 3$, $\log_x 81$, $\log_x 2187$. Find the sum to n terms, giving your answer in the form $k\log_x 3$, where k is in terms of n. [3]

(b) The first 3 terms of a geometric progression are 1, $3\tan^2\theta$, $9\tan^4\theta$, for $0 < \theta < \frac{\pi}{2}$.

Find the values of θ for which this geometric progression has a sum to infinity. [4]

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4(a)	$d = 3\log_x 3$ or $\log_x 3^3$ nfww	**B1**	Must be exact Allow $d = \log_x 27$ nfww
	$\frac{n}{2}\left(2\log_x 3 + 3(n-1)\log_x 3\right)$ nfww	**M1**	For use of sum formula with *their d* must be in the form of $\log_x 3$
	$\frac{n}{2}(3n-1)\log_x 3$ or $\left(\frac{3n^2}{2} - \frac{n}{2}\right)\log_x 3$	**A1**	Must be in the form of $k\log_x 3$
4(b)	$r = 3\tan^2\theta$	**B1**	soi
	$\lvert 3\tan^2\theta\rvert < 1$ or $[-1 <]3\tan^2\theta < 1$ or $\left[-\frac{1}{3} <\right]\tan^2\theta < \frac{1}{3}$ or $[0 <]\tan^2\theta < \frac{1}{3}$	**B1**	
	$\tan\theta < \frac{1}{\sqrt{3}}$ or $0 < \tan\theta < \frac{1}{\sqrt{3}}$	**B1**	
	$0 < \theta < \frac{\pi}{6}$	**B1**	

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5 The first term of a geometric progression is 10.
This geometric progression has a positive common ratio r.

The first term of an arithmetic progression is also 10.
This arithmetic progression has a negative common difference d.

The second term of the geometric progression is the same as the fourth term of the arithmetic progression.
The third term of the geometric progression is the same as the sixth term of the arithmetic progression.

(a) Find the values of r and d. [6]

(b) Determine whether the geometric progression has a sum to infinity. [1]

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5(a)	$10r = 10 + 3d$ soi	**B1**	
	$10r^2 = 10 + 5d$ soi	**B1**	
	$10\left(\frac{10+3d}{10}\right)^2 = 10 + 5d$ oe or $10r^2 = 10 + \frac{5(10r-10)}{3}$ oe	**M1**	
	$9d^2 + 10d = 0$ or $3r^2 - 5r + 2 = 0$ oe	**A1**	
	Correct method to find d or r	**A1**	**FT** *their* quadratic in d or r
	$d = -\frac{10}{9}$ and $r = \frac{2}{3}$	**A1**	
5(b)	Appropriate determination for *their r* e.g. $\lvert r \rvert < 1$ so the geometric progression has a sum to infinity oe	**B1**	**STRICT FT** *their r*

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6 A geometric progression has a 4th term of $\frac{8k^6}{27}$ and a 6th term of $\frac{32k^{10}}{243}$, where k is a constant.

The common ratio of this geometric progression is positive.

(a) Find the common ratio in terms of k and the value of the first term of this geometric progression. [4]

(b) Given that this geometric progression has a sum to infinity of 3, find the possible values of k. [3]

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6(a)	$ar^3 = \frac{8k^6}{27}$, $ar^5 = \frac{32k^{10}}{243}$ soi	**B1**	May be implied by a correct 5th term $\frac{16k^8}{81}$
	$r = \frac{2k^2}{3}$	**2**	**M1** for solution of *their* equations to obtain either $r = mk^2$ or $a = c$, where m is an unsimplified numeric constant and c is a non-zero constant e.g. Use of $r^2 = \frac{6\text{th term}}{4\text{th term}}$
	$a = 1$	**A1**	
6(b)	$\frac{1}{1-\frac{2k^2}{3}} = 3$ soi	**B1**	For use of sum to infinity formula with *their* values of a where a is numeric and r which is in terms of k^2
	$k = \pm 1$	**2**	**B1** for each must be from correct work

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7 **(a)** An arithmetic progression has twelve terms. The sum of the first three terms is -36 and the sum of the last three terms is 72. Find the first term and the common difference. [5]

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(b) The first three terms of a geometric progression are 1, 1.2 and 1.44. Find the smallest value of n such that the sum of the first n terms is greater than 500. [5]

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7(a)	Correct pair of simplified linear equations in a and d with terms collected, e.g., $3a+3d=-36$ isw or $a+d=-12$ isw $3a+30d=72$ isw or $a+10d=24$ isw	**B3**	**B2** for one correct simplified equation or **B1** for $a+a+d+a+2d=-36$ or $\frac{3}{2}\{2a+(3-1)d\}=-36$ or $a+9d+a+10d+a+11d=72$ or $\frac{12}{2}\{2a+(12-1)d\}-\frac{9}{2}\{2a+(9-1)d\}=72$ or $12a+66d-9a-36d=72$ or $\frac{3}{2}\{2(a+9d)+(3-1)d\}=72$
	Solves two linear equations for d or a e.g. $27d=108 \rightarrow d=\ldots$ or $9d=36 \rightarrow d=\ldots$ or $a+10(-12-a)=24 \rightarrow a=\ldots$ $27a=-432 \rightarrow a=\ldots$	**M1**	**FT** *their* linear equations in a and d providing at least **B1** earned and the equations have a solution
	$d=4$ and $a=-16$ nfww	**A1**	
7(b)	1.2^n *101	**B3**	where * is any inequality sign or =; **B2** for $\frac{[1](1.2^n-1)}{(1.2-1)}*500$ or **B1** for $r=1.2$ soi
	$n\log 1.2*\log 101$ or $\log_{1.2}101$ soi	**M1**	**FT** 1.2^n * *their* 101 providing **B2** has been awarded and (*their* 101) >0
	$n=26$	**A1**	**dep** on all previous marks awarded

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8 **(a)** A geometric progression has first term 64 and common ratio 0.5.

(i) Find the 10th term. [2]

(ii) Find the sum of the first 10 terms. [2]

(iii) Find the sum to infinity. [1]

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(b) An arithmetic progression is such that $S_{20} - 400 = 2S_{10}$ and $u_1 : u_6$ is 1 : 5. Find the sum of the first 3 terms of this progression. [6]

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8a)(i)	$\frac{1}{8}$ or 0.125	**2**	**M1** for $64(0.5)^9$ oe
8(a)(ii)	$\frac{1023}{8}$ or 127.875	**2**	**M1** for $\frac{64(1-0.5^{10})}{1-0.5}$ oe
8(a)(iii)	128	**1**	
8(b)	$\frac{20}{2}\{2a+19d\}-400=2\times\frac{10}{2}\{2a+9d\}$ oe, soi	**M2**	**M1** for $\frac{20}{2}\{2a+19d\}$ or $\frac{10}{2}\{2a+9d\}$ soi
	$5a = a + 5d$ soi	**M1**	
	$d = 4$	**A1**	
	$a = 5$	**A1**	
	27 nfww	**B1**	must have earned all previous marks

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9 An arithmetic progression has first term a and common difference d. The third term is 13 and the tenth term is 41.

(a) Find the value of a and of d. [4]

(b) Find the number of terms required to give a sum of 2555. [4]

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(c) Given that S_n is the sum to n terms, show that $S_{2k} - S_k = 3k(1+2k)$. [4]

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9(a)	Attempts to solve $a + 2d = 13$ and $a + 9d = 41$ oe	**M2**	**M1** for $a + 2d = 13$ and $a + 9d = 41$ soi
	$d = 4$ and $a = 5$	**A2**	**A1** for $d = 4$ or $a = 5$
9(b)	$\frac{n}{2}\{2(5)+(n-1)4\}$ soi	**M1**	**FT** *their a* and *their d*
	$2n^2+3n-2555\,[*0]$	**A1**	where * could be = or any inequality sign
	Solves *their* 3-term quadratic of the form ax^2+bx+c [*0] by factorising or formula or *their* 3-term quadratic of the form ax^2+bx*c or better if completing the square	**M1**	
	35	**A1**	

9(c)	May work ***consistently*** in *n* throughout but must conclude in *k* to earn the final mark		
	$S_{2k}=\frac{2k}{2}\{10+(2k-1)4\}$ soi	**B1**	**FT** *their a* and *their d*
	$\frac{2k}{2}\{10+(2k-1)4\}-\frac{k}{2}\{10+(k-1)4\}$ soi	**M1**	**FT** *their a* and *their d* ; condone at most one error
	Simplifies as far as e.g. $8k^2 + 6k - (3k + 2k^2)$ or $8k^2 + 6k - 3k - 2k^2$	**A1**	
	Correct completion to given answer: $6k^2 + 3k = 3k(1 + 2k)$	**A1**	
	Alternative method		
	$\frac{2k}{2}\{2a+(2k-1)d\}$ **and** *a* = *their* 5 and *d* = *their* 4 substituted at some point	**(B1)**	
	$ak-\frac{d}{2}k+\frac{3}{2}dk^2$ oe	**(M1)**	condone at most one error
	$5k-\frac{4}{2}k+\frac{3}{2}\times4\times k^2$	**(A1)**	
	Correct completion to given answer: $6k^2 + 3k = 3k(1 + 2k)$	**(A1)**	

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10 An arithmetic progression, A, has first term a and common difference d.
The 2nd, 14th and 17th terms of A form the first three terms of a convergent geometric progression, G, with common ratio r.

(a) **(i)** Given that $d \neq 0$, find two expressions for r in terms of a and d and hence show that $a = -17d$. [6]

(ii) Find the value of r. [2]

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(b) The first term of the geometric progression, G, is q and the sum to infinity is $\frac{256}{3}$.

Find the sum of the first 20 terms of the **arithmetic** progression, A. [7]

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10(a)(i)	$a + d$, $a + 13d$, $a + 16d$ soi	**B1**	
	$\frac{a+13d}{a+d} = \frac{a+16d}{a+13d}$ oe	**M2**	**FT** *their* 3 distinct terms providing of the form $a + kd$ where $k \neq 0$ and at least one is correct **M1** for either $[r =]\frac{a+13d}{a+d}$ or $[r =]\frac{a+16d}{a+13d}$ or $[r =]\sqrt{\frac{a+16d}{a+d}}$
	Clears fractions and expands oe: $a^2 + 26ad + 169d^2$ $= a^2 + 17ad + 16d^2$	**A1**	
	$9ad + 153d^2 = 0$ or $9ad = -153d^2$	**A1**	
	Convincingly derives $a = -17d$ e.g. $9d(a + 17d) = 0$ [therefore] $a = -17d$ oe	**A1**	

10(a)(ii)	$r = 0.25$ oe	**2**	**M1** $\frac{-17d+13d}{-17d+d}$ or $\frac{-17d+16d}{-17d+13d}$ or $-16d$, $-4d$, $-d$
10(b)	$\frac{q}{1-0.25} = \frac{256}{3}$ oe	**M1**	**FT** *their* 0.25 providing it is between -1 and 1
	$q = 64$	**A1**	
	$[a + d =$ *their* 64 $]$ $-17d + d =$ *their* 64 or $a - \frac{a}{17} =$ *their* 64	**DM1**	**dep** on previous **M1**
	$d = -4$ and $a = 68$ oe OR $d = -4$ and $S_{20} = -150d$ oe	**A2**	**A1** for either correct
	$S_{20} = \frac{20}{2}\{2(their 68) + 19(their(-4))\}$ or $S_{20} = -150(their\ d)$	**M1**	**FT** *their a* and *d*
	600	**A1**	

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