Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Binomial expansion

Time allowed: 1 hour 30 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 2, 4-7 (Paper 1): no calculator.
Questions 1, 3, 8-10 (Paper 2): calculator allowed.

Total marks

/ 57

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 57 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 Using the binomial theorem, expand $(1+e^{2x})^4$, simplifying each term. [2]

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1	$1+4e^{2x}+6e^{4x}+4e^{6x}+e^{8x}$	**B2**	mark final answer for **B2** **B1** for any 3 correct simplified terms in a sum or all 5 simplified terms listed but not summed or for a correct, simplified expansion that is not their final answer or **M1** for a correct unsimplified expansion e.g. $1+4e^{2x}+\frac{4\times3}{2}(e^{2x})^2+\frac{4\times3\times2}{6}(e^{2x})^3+(e^{2x})^4$ If 0 scored, **SC1** for a complete, correct, simplified expansion as final answer found by multiplying out the brackets

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2 **(a)** In the expansion of $(1+kx)^{15}$, where k is a constant, the coefficient of x^3 is $-29\,120$. Find the value of k. [2]

(b) Find the term independent of y in the expansion of $\left(8y^2-\frac{1}{2y}\right)^{12}$. [2]

Binomial expansion Click for more variations

2(a)	${}^{15}\mathrm{C}_3k^3=-29120$ oe	**M1**	
	$k=-4$	**A1**	

2(b)	${}^{12}C_8\left(8y^2\right)^4\left(-\frac{1}{2y}\right)^8$ or ${}^{12}C_4\left(8y^2\right)^4\left(-\frac{1}{2y}\right)^8$	**M1**	
	7920	**A1**	

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3 **(i)** Find, in the simplest form, the first 3 terms of the expansion of $(2-3x)^6$, in ascending powers of x. [3]

(ii) Find the coefficient of x^2 in the expansion of $(1+2x)(2-3x)^6$. [2]

Binomial expansion Click for more variations

3 **(i)**	$(2-3x)^6 = 64 - 576x + 2160x^2$ isw	B1B1B1	
(ii)	$2160 - 2 \times 576 = 1008$	M1 A1	*their* final 2160 + 2 × *their* final –576

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4 **(i)** Find the first 3 terms, in descending powers of x, in the expansion of $\left(x + \frac{2}{x^2}\right)^6$. [3]

(ii) Hence find the term independent of x in the expansion of $\left(2 - \frac{4}{x^3}\right)\left(x + \frac{2}{x^2}\right)^6$. [2]

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4 **(i)** $\left(x+\frac{2}{x^2}\right)^6 = x^6 + 12x^3 + 60...$ **(ii)** Independent term = $(2\times'60') + (-4\times'12') = 72$	B3 [3] M1 A1 [2]	B1 for each correct term M1 for sum of 2 products (2 × their 60)+(–4 × their 12) A1 for 72

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5 Find the coefficient of x^2 in the expansion of $\left(x-\frac{3}{x}\right)\left(x+\frac{2}{x}\right)^5$. [5]

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5	$x^5 + 10x^3 + 40x + ...$	**3**	**M1** for attempt to expand $\left(x + \frac{2}{x}\right)^5$, with at least 2 correct terms **A1** for $10x^3$ **A1** for $40x$
	Term in x^2 : $(1\times 40) - (3\times 10)$	**M1**	For $(1\times \textit{their } 40) \pm (3\times \textit{their } 10)$
	10	**A1**	

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6 The coefficient of x^2 in the expansion of $(2 + px)^6$ is 60.

(i) Find the value of the positive constant p. [3]

(ii) Using your value of p, find the coefficient of x^2 in the expansion of $(3 - x)(2 + px)^6$. [3]

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6	**(i)** $^6C_2\,(2^4)\,(px)^2$ or $\binom{6}{2}2^4\,(px)^2$	B1	Seen or implied, unsimplified
	$240p^2 = 60$	M1	M1 for their coefficient of $x^2 = 60$ and attempt to solve
	$p = \frac{1}{2}$	A1 [3]	
	(ii) coefficients of the terms needed	M1	M1 for realising that 2 terms are involved
	$(-1)\,{}^6C_1\,(2)^5\,p + (3 \times 60)$	B1	B1 for $(-1)\,{}^6C_1\,(2)^5\,p$ or $-192p$, using their p.
	$= 84$	A1 [3]	

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7 **(i)** Find, in ascending powers of x, the first 3 terms in the expansion of $\left(2-\frac{x^2}{4}\right)^6$. Give each term in its simplest form. [3]

(ii) Hence find the coefficient of x^2 in the expansion of $\left(2-\frac{x^2}{4}\right)^6\left(\frac{1}{x}+x\right)^2$. [4]

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7(i)	$64-48x^2+15x^4$	**B3**	**B1** for each correct term in final line of response
7(ii)	$\left(64-48x^2+15x^4\right)\left(\frac{1}{x^2}+2+x^2\right)$	**B1**	**B1**for $\frac{1}{x^2}+2+x^2$ oe
	at least two correctly obtained products leading to terms in x^2	**M1**	
	Term in x^2 : $64+15-96$	**A1**	**FT** for correct evaluation of *their* 64 + (2 × *their*–48) + *their* 15
	$=-17$	**A1**	

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8 The first three terms in the expansion of $(a+bx)^5(1+x)$ are $32-208x+cx^2$. Find the value of each of the integers a, b and c. [7]

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8	$a^5 + 5a^4bx + 10a^3b^2x^2$	**2**	**B1** for powers or for coefficients
	$a^5 + \left(a^5 + 5a^4b\right)x + \left(10a^3b^2 + 5a^4b\right)x^2$	**2**	**M1** for multiplying to obtain 5 terms **A1** for all correct
	$a^5 = 32 \rightarrow a = 2$	**A1**	
	$32 + 80b = -208 \rightarrow b = -3$	**A1**	
	$10 \times 8 \times 9 + 5 \times 16 \times -3 = c \rightarrow c = 480$	**A1**	

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9 **(a)** In the expansion of $\left(x+x^2\right)^8$ in ascending powers of x, the 3rd and 6th terms are equal.

Find the value of x. [3]

(b) In the expansion of $\left(x+\frac{2}{x}\right)^n$ in decreasing powers of x, the 6th term is a constant.

(i) Find the value of the positive integer n. [2]

(ii) Find the value of the 6th term. [2]

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9(a)	$x = \sqrt[3]{\frac{28}{56}}$ oe, nfww, isw	**3**	**B2** for $28x^{10}$ and $56x^{13}$ OR $[x^8](28x^2)$ and $[x^8](56x^5)$ or **B1** for $28x^{10}$ or $56x^{13}$ OR $[x^8](28x^2)$ or $[x^8](56x^5)$
9(b)(i)	$n = 10$	**2**	**B1** for the correct term in any form e.g. ${}^nC_5x^{n-5}\left(\frac{2}{x}\right)^5$ or for $n - 5 = 5$ oe, soi
9(b)(ii)	8064	**2**	**B1** for ${}^{10}C_5 \times 2^5$ oe e.g. 252×32

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10 The first three terms, in descending powers of x, in the expansion of $\left(3x^2 - a\right)^n \left(1 + \frac{1}{x^2}\right)^2$ can be written as $729x^{12} + 972x^{10} + bx^8$, where a, b and n are constants.

Find the values of a, b and n. [9]

Binomial expansion Click for more variations

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10	$n = 6$ stated; nfww	**B2**	**B1** for first term: $\left(3x^2\right)^n$ or $\left(3x^2\right)^6$ soi
	$-6\left(3x^2\right)^5 a + 2\left(729x^{10}\right) = 972x^{10}$ oe and $\frac{6\times5\left(3x^2\right)^4(-a)^2}{2} + 6\left(3x^2\right)^5 \times \frac{2}{x^2}(-a) + 729x^8 = bx^8$ oe	**B3**	**B2** for one correct equation **OR** **B1** for second term: $n(-a)\left(3x^2\right)^{n-1}$ or $6(-a)\left(3x^2\right)^5$ or $-1458ax^{10}$ soi **B1** for third term: $\frac{n(n-1)(-a)^2\left(3x^2\right)^{n-2}}{2}$ or $\frac{6(5)(-a)^2\left(3x^2\right)^4}{2}$ or $1215a^2x^8$ soi **B1** for $1 \ [+] \ \frac{2}{x^2} \ [+] \ \frac{1}{x^4}$ soi
	$a = \frac{1}{3}$	**B2**	**B1** for $-1458a + 1458 = 972$ soi
	$b = -108$	**B2**	**B1** for $1215a^2 - 2916a + 729$ soi

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