Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Connected rates of change and small changes

Time allowed: 1 hour 15 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 2, 9 (Paper 1): no calculator.
Questions 1, 3-8, 10 (Paper 2): calculator allowed.

Total marks

/ 50

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 50 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 **(i)** Given that $y = \sin 3x$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [1]

(ii) Hence find the approximate increase in y as x increases from $\frac{\pi}{9}$ to $\frac{\pi}{9} + p$, where p is small. [2]

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1	**(i)** $\frac{\mathrm{d}y}{\mathrm{d}x} = 3\cos 3x$	B1
	(ii) Uses $\partial y = \left(\text{attempt at } \frac{\mathrm{d}y}{\mathrm{d}x}\right) \times \partial x$ with $x = \frac{\pi}{9}$ and $\partial x = p$.	M1
	$1.5p$	A1√ on k [3]

2 The rate of change of a variable x with respect to time t is $4\cos^2 t$.

(i) Find the rate of change of x with respect to t when $t = \frac{\pi}{6}$. [1]

The rate of change of a variable y with respect to time t is $3\sin t$.

(ii) Using your result from part **(i)**, find the rate of change of y with respect to x when $t = \frac{\pi}{6}$. [3]

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2	**(i)** 3 or $\frac{3}{1}$	B1 [1]	
	(ii) $\frac{dy}{dx}=\frac{3\sin t}{4\cos^2 t}$ $\left(=\frac{3\sin t}{3}\right)$	M1	M1 correct substitution in $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ o.e.
	$=\frac{3\sin\frac{\pi}{6}}{3}$	DM1	
	$= 0.5$	A1 [3]	DM1 for use of their '3' and substitution of $\frac{\pi}{6}$.

3 The total surface area, A cm², of a solid cylinder with radius r cm and height 5 cm is given by $A = 2\pi r^2 + 10\pi r$. Given that r is increasing at a rate of $\frac{0.2}{\pi}$ cm s^{-1}, find the rate of increase of A when r is 6. [4]

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3	$\left(\frac{dA}{dr}=\right)4\pi r+10\pi$	B1,B1	
	Use $\frac{dA}{dt}=\frac{dA}{dr}\times\frac{dr}{dt}$ with $r = 6$	M1	Their $\frac{dA}{dr}$
	6.8	A1 [4]	Rounds to 6.8

4 Variables x and y are such that $y = \cos x \sin^2 x$. Use differentiation to find the approximate change in y as x increases from 3 to $3 + h$, where h is small. [5]

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4	$\frac{\mathrm{d}}{\mathrm{d}x}(\sin^2 x) = 2\sin x \cos x$ soi	**B1**	
	$\cos x \times their(2\sin x \cos x) +$ $(their - \sin x) \times \sin^2 x$	**M1**	**FT** *their* $\frac{\mathrm{d}}{\mathrm{d}x}(\sin^2 x)$
	$\cos x \times their(2\sin x \cos x) + (-\sin x) \times \sin^2 x$ isw	**A1**	**FT** *their* $\frac{\mathrm{d}}{\mathrm{d}x}(\sin^2 x)$
	$\frac{\delta y}{h} = their(2\sin 3\cos^2 3 - \sin^3 3)$ or better	**M1**	**FT** *their* derivative
	$0.274h$ or $0.2738[08...]h$ where the coefficient of h is rot to 4 or more sf	**A1**	**dep** on correct derivative seen

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5 Variables x and y are related by the equation $y = 1 + \frac{2}{x} + \frac{1}{x^2}$ where $x > 0$. Use differentiation to find the approximate change in x when y increases from 4 by the small amount 0.01. [5]

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5	Substitutes $y = 4$ and rearranges to correct 3-term quadratic $3x^2 - 2x - 1 = 0$ oe	**B1**	
	Solves *their* 3-term quadratic in x as far as $x = \ldots$	**M1**	
	$\frac{dy}{dx} = -2x^{-2} - 2x^{-3}$ oe, isw	**B1**	
	$\frac{0.01}{\delta x} = their\left(\left.\frac{dy}{dx}\right\|_{x=\ their\ 1}\right)$ or better	**M1**	**FT** *their* derivative and *their x,* providing *their* $x > 0$ and *their* $x \neq 4$ unless 4 is a genuine solution of *their* 3-term quadratic; must see a power decrease for attempted differentiation in two out of the three terms
	$-\frac{1}{400}$ oe as the only solution	**A1**	**dep** on all previous marks being awarded

5	**Alternative method** $x = \frac{1}{\sqrt{y} - 1}$ or $x = \frac{1 + \sqrt{y}}{y - 1}$ or $x = \frac{-1 - \sqrt{y}}{1 - y}$ oe	**(B2)**	**B1** for $y = \frac{(x+1)^2}{x^2}$ or better or for $x = \frac{2 + \sqrt{4 - 4(y-1)(-1)}}{2(y-1)}$ or for $x = \frac{-2 - \sqrt{4 - 4(1-y)}}{2(1-y)}$
	$\frac{dx}{dy} = -\left(\sqrt{y} - 1\right)^{-2}\left(\frac{1}{2}y^{-\frac{1}{2}}\right)$ oe isw or $\frac{dx}{dy} = \frac{(y-1)\left(\frac{1}{2}y^{-\frac{1}{2}}\right) - (1 + \sqrt{y})}{(y-1)^2}$ oe	**(M1)**	or $\frac{dx}{dy} = \frac{(1-y)\left(-\frac{1}{2}y^{-\frac{1}{2}}\right) - (-1 - \sqrt{y})(-1)}{(1-y)^2}$ oe
	$\frac{\delta x}{0.01} = their\left(\left.\frac{dx}{dy}\right\|_{y\ =\ 4}\right)$ or better	**(M1)**	**FT** *their* derivative ; must have attempted derivative
	$-\frac{1}{400}$ oe as the only solution	**(A1)**	**dep** on all previous marks being awarded

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6 A cylinder, open at both ends, has base radius r cm and height $4r$ cm. Its curved surface area is S cm^2.

Given that r varies with time t, find S at the instant when $\frac{\mathrm{d}S}{\mathrm{d}t} = 6\frac{\mathrm{d}r}{\mathrm{d}t}$. [5]

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6	$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$ or $\frac{dS}{dr} = 6$ soi	**B1**	
	$S = 2\pi r(4r)$ or $8\pi r^2$	**B1**	
	$16\pi r = 6$	**M1**	**FT** *their* $S = k\pi r^2$ with k a positive integer to give $2k\pi r = 6$
	$r = \frac{6}{16\pi}$ oe, isw	**A1**	
	$S = \frac{9}{8\pi}$ oe, isw	**A1**	

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7 Variables x and y are such that $y = \dfrac{(1+\sin 3x)^4}{\sqrt{x}}$. Use differentiation to find the approximate change in y when x increases from 1.9 to $1.9+h$, where h is small. [6]

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7	$\frac{\mathrm{d}}{\mathrm{d}x}(\sin 3x) = 3\cos 3x$ soi	**B1**	
	$u = (1+\sin 3x)^4$ $\frac{\mathrm{d}u}{\mathrm{d}x} = 4(1+\sin 3x)^3(3\cos 3x)$ soi	**M1**	**FT** *their* $3\cos 3x$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{x}\left(4(1+\sin 3x)^3(3\cos 3x)\right) - (1+\sin 3x)^4 \times \frac{1}{2}x^{-\frac{1}{2}}}{\left(\sqrt{x}\right)^2}$	**M1**	FT *their* $\frac{\mathrm{d}u}{\mathrm{d}x}$ *or their* $\frac{\mathrm{d}v}{\mathrm{d}x}$ but not both
	Correct derivative	**A1**	
	Evaluates *their* derivative at $x = 1.9$ and multiplies by h	**M1**	
	$0.651h$	**A1**	

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8 A cube of side x cm has surface area $S\,\text{cm}^2$. The volume, $V\,\text{cm}^3$, of the cube is increasing at a rate of $480\,\text{cm}^3\text{s}^{-1}$. Find, at the instant when $V = 512$,

(a) the rate of increase of x, [4]

(b) the rate of increase of S. [2]

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8(a)	$\left[\frac{\mathrm{d}(x^3)}{\mathrm{d}x}=\right]3x^2$ and $x=\sqrt[3]{512}$ soi OR $\left[\frac{\mathrm{d}(\sqrt[3]{V})}{\mathrm{dV}}=\right]\frac{1}{3}V^{-\frac{2}{3}}$	**B1**	
	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}x}{\mathrm{d}V}$ oe, soi	**B1**	
	$\frac{480}{3(8)^2}$ oe	**M1**	**FT** *their* $\frac{\mathrm{d}V}{\mathrm{d}x}=k(8)^2$ or $\frac{\mathrm{d}x}{\mathrm{d}V}=k(512)^{-\frac{2}{3}}k\neq 0$
	2.5 oe	**A1**	
8(b)	$12(8)\times$ *their* 2.5 soi	**M1**	**FT** *their* 8 provided it is not 512
	240	**A1**	**FT** provided at least M1 earned in **(a)**

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9 It is given that $y = \dfrac{\ln(3x^2 - 1)}{x + 2}$, for $x > \dfrac{1}{\sqrt{3}}$. When $x = 1$, y is increasing at the rate of h units per second. Find, in terms of h, the corresponding rate of change in x, giving your answer in exact form.

[6]

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9	$\left(\frac{dy}{dx}=\right)\frac{(x+2)\left(\frac{6x}{3x^2-1}\right)-\ln\left(3x^2-1\right)}{(x+2)^2}$ or $\frac{6x}{\left(3x^2-1\right)}(x+2)^{-1}-(x+2)^{-2}\ln\left(3x^2-1\right)$ oe	**3**	**B1** for $\frac{6x}{3x^2-1}$ **M1** for correct attempt at differentiation of a quotient or a correct product **A1** for all terms apart from $\frac{6x}{3x^2-1}$ correct.
	When $x = 1$, $\frac{dy}{dx}=\frac{9-\ln 2}{9}$	**M1**	For use of $x = 1$ in *their* $\frac{dy}{dx}$, must see a substitution if in decimal form unless 0.923 obtained from a correct derivative
	$\frac{dx}{dt}=\frac{9h}{9-\ln 2}$ or exact equivalent	**2**	**M1** for $\frac{h}{their\left(\frac{9-\ln 2}{9}\right)}$, with $x = 1$ substituted in **A0** if using small changes

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10 Variables x and y are related by the equation $y = 2 + \tan(1 - x)$ where $0 \leqslant x \leqslant \frac{\pi}{2}$. Given that x is increasing at a constant rate of 0.04 radians per second, find the corresponding rate of change of y when $y = 3$. [6]

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10	$\frac{dy}{dx} = -\sec^2(1-x)$ oe	**B2**	**must be seen** **B1** for $\frac{dy}{dx} = k\sec^2(1-x)$ oe, $k \neq -1$ or **SC1** for $\frac{dy}{dx} = -\sec^2 1-x$ or $\frac{dy}{dx} = -\sec^2(1-x)+c$
	Solves $3 = 2 + \tan(1-x)$ as far as $1-x = \tan^{-1}1$	**M1**	
	$1-x = \frac{\pi}{4}$ isw or 0.7853[98…] $x = 1-\frac{\pi}{4}$ isw or 0.2146[01…]	**A1**	
	Correct use of chain rule **and** correctly writes in terms of cosine or tangent: $\frac{their(-1)}{\cos^2\left(their\frac{\pi}{4}\right)} \times 0.04$ oe, soi or $their(-1)\left\{1+\tan^2\left(their\frac{\pi}{4}\right)\right\} \times 0.04$ oe soi	**M1**	**dep** on at least **B1** and an attempt to solve $3 = 2 + \tan(1-x)$
	–0.08 oe, nfww	**A1**	**dep** on all previous marks awarded

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