Mark scheme

Coordinate geometry of the circle

Additional Mathematics 0606 · Questions from Paper 1 and Paper 2

Total marks

64

MARKS FOR EACH QUESTION

1	2	3	4	5	6	7	8	9	10	Total
4	5	5	6	6	5	7	8	8	10	**64**

Question 1 **[4]**

1	$(x-2)^2+(y+1)^2=74$ oe nfww or $x^2+y^2-4x+2y-69=0$ oe nfww	**4**	**M1** for centre: $\left(\frac{-3+7}{2},\frac{6-8}{2}\right)$ or $(2,-1)$ soi **M2** for [r =] $\sqrt{74}$ or [r^2 =] 74 or [diameter =] $2\sqrt{74}$ or $c=-69$ or **M1** for $\left[AB^2=\right](-3-7)^2+(6--8)^2$ oe or $\left[r^2=\right](-3-their2)^2+(6-their(-1))^2$ oe or $\left[r^2=\right](7-their2)^2+(-8-their(-1))^2$ oe **Alternative method** **M2** for $\frac{y-6}{x-(-3)}\times\frac{y-(-8)}{x-7}=-1$ or **M1** for e.g. $P(x, y)$ on the circumference means $AP\perp BP\rightarrow m_{AP}\times m_{BP}=-1$ **M1** for $\frac{y^2+2y-48}{x^2-4x-21}=-1$ oe **FT** $their\left(\frac{y-6}{x-(-3)}\times\frac{y-(-8)}{x-7}\right)$ with at most one sign error

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Question 2 **[5]**

2	$(2y+1)^2 + y^2 = 29$ (or $5x^2 - 2x - 115 = 0$)	M1	M1 for attempt to get an equation in terms of one variable only
	leading to $5y^2 + 4y - 28 = 0$	DM1	DM1 for obtaining a 3 term quadratic equation
	(or $x^2 + \left(\frac{x-1}{2}\right)^2 = 29$)	DM1	DM1 for attempt to solve quadratic equation
	$x = -\frac{23}{5}, y = -\frac{14}{5}$ and	A1	A1 for a pair of values
	$x = 5, y = 2$	A1	
	(5, 2) spotted gets B1	**[5]**	

Question 3 **[5]**

3(a)	$(x-4)^2 + (y-2)^2 = 40$ oe, isw or $x^2 + y^2 - 8x - 4y - 20 = 0$ oe, isw	**B2**	**B1** for $r^2 = 40$ or $r = \sqrt{40}$ oe, soi
3(b)	$16 + (y-2)^2 = 40$ oe	**M1**	**FT** *their* equation of the circle providing it is the equation of a circle
	$(y-2)^2 = 24$ or $y^2 - 4y - 20 = 0$	**A1**	Obtain correct quadratic equation in any form
	$(0, 2 \pm \sqrt{24})$ isw or equivalent exact form	**A1**	

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Question 4 **[6]**

4(a)	Grad $AB = \frac{6}{8}$ oe Grad $AC = -\frac{2}{14}$ oe Grad $BC = -\frac{8}{6}$ oe	**2**	**B1** for 2 correct unsimplified
4(b)	Centre $(5,\ 3)$	**B1**	**FT** on *their* diameter, allow for $(2,\ 7)$ or $(9,\ 6)$
	Radius^2 = 50 oe soi	**2**	**M1** for use of *their* centre and coordinates of either A, B or C to find the square of the radius or the radius. Centre must not be A, B or C. or Finding the length between any 2 points A, B or C **and** halving it.
	$(x-5)^2+(y-3)^2=50$ oe	**A1**	Allow $\left(5\sqrt{2}\right)^2$ for radius^2

Question 5 **[6]**

5(a)	Centre $(5,\ 12)$ soi	**B1**	
	Gradient of radius = $\frac{12-4}{5-11}$ $\left(=-\frac{4}{3}\right)$	**M1**	
	Equation of normal: $y-4=\frac{3}{4}(x-11)$	**M1**	Must be using *their* perpendicular gradient and the correct point
	$3x-4y=17$	**A1**	
5(b)	Distance between centres = 13 Sum of radii = 12	**M1**	For both
	$13>12$ so no intersection	**A1**	

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Question 6 [5]

6	Radius: 5 soi	**B1**	Implied by $r^2 = 25$ or 25 used as r^2 in *their* 2nd circle equation
	Centre: $\left(4\sqrt{2}, 4\sqrt{2}\right)$ soi oe	**B2**	**B1** for (5.66, 5.66) or better OR **B1** for distance between centres is 8 soi
	$\left(x-\frac{8}{\sqrt{2}}\right)^2+\left(y-\frac{8}{\sqrt{2}}\right)^2 = 25$ oe or $x^2+y^2-2\left(4\sqrt{2}\right)x-2\left(4\sqrt{2}\right)y+2\left(4\sqrt{2}\right)^2-25=0$ oe	**M1**	**FT** $their\left(4\sqrt{2}, 4\sqrt{2}\right)$ in form (k, k) where k is a non-zero constant e.g. $\left(x-their\frac{8}{\sqrt{2}}\right)^2+\left(y-their\frac{8}{\sqrt{2}}\right)^2 = 25$ or $x^2+y^2-2theirx-theiry$ $+2(their)^2-25=0$ oe
	$x^2+y^2-8\sqrt{2}\,x-8\sqrt{2}\,y+39=0$	**A1**	Must be exact

Question 7 [7]

7(a)	Centre (2, 1)	**B1**	Accept $x = 2$ and $y = 1$
	Radius 2 soi	**B1**	
	$(x-2)^2+(y-1)^2=4$ oe	**B1**	**FT** *their* centre and radius ISW Do not accept 2^2 for 4
7(b)	$(x-2)^2+(2x+a-1)^2=4$	**M1**	Substitute $y = 2x + a$ into *their* circle equation
	$x^2-4x+4+4x^2+4(a-1)x+(a-1)^2=4$	**A1**	Equivalent must be seen as **AG**
	leading to $5x^2+4(a-2)x+(a-1)^2=0$ **AG**		
	$16(a-2)^2-20(a-1)^2=0$	**M1**	for finding the discriminant of the given equation and equating to zero
	$a^2+6a-11=0$ $a=-3\pm\sqrt{20}$ or $\frac{-6\pm\sqrt{80}}{2}$ oe	**A1**	isw from a correct answer isw

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Question 8 **[8]**

8(a)	$5x^2-30x+45\ (=0)$ or equivalent 3 term quadratic	**2**	**M1** for attempt to eliminate one variable
	Use of discriminant or solution to show one repeated root or a single solution	**A1**	Must have conclusion e.g. Discriminant = 0 (so tangent) oe One solution only (so tangent)
	Alternative 1		
	Centre of the circle (5, 2) Equation of radius perpendicular to given line $y-2=-\frac{1}{2}(x-5)$ Intersection at $(3,3)$	**(2)**	**M1** for a complete method to obtain the point of intersection
	Show that P lies on the circle or that $CP=\sqrt{5}$	**(A1)**	
	Alternative 2		
	The line $y=kx+n$ touches the circle $(x-p)^2+(y-q)^2=r^2$ if $(-kp+q-n)^2=r^2(1+k^2)$ so $(-(2\times5)+2-3)^2=5(1+2^2)$	**(2)**	**M1** for a complete method showing substitution, allow one slip
	25 = 25, so a tangent	**(A1)**	

8(b)	(3, 3)	**2**	**B1** for each
8(c)	Radius = $3\sqrt{2}$ oe soi	**B1**	
	$(x-3)^2+(y-3)^2$ soi	**B1**	**FT** on *their* answer to **(b)** only
	$(x-3)^2+(y-3)^2=18$ oe	**B1**	ISW any expansion

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Question 9 **[8]**

9 (i)	$(x+2)^2 + x^2 = 10$	**B1**	
	$x^2 + 2x - 3 = 0 \rightarrow (x+3)(x-1) = 0$	**M1**	3 term quadratic with attempt to solve
	Points (1, 3), (–3, –1) isw	**A1**	both x or a pair
		A1	both y or second pair
	or elimination of x leads to $y^2 - 2y - 3 = 0$, then as above		
(ii)	$m^2x^2 + 10mx + 25 + x^2 = 10$	**B1**	
	$(m^2+1)x^2 + 10mx + 15 = 0$		
	$b^2 - 4ac = (0) \rightarrow 100m^2 - 60(m^2+1) = 0$	**M1**	attempt to use discriminant on three term quadratic. Allow unsimplified
		A1	
	$m = \pm\sqrt{\frac{3}{2}}$ oe isw	**A1**	cao ± is required
	Alternative solution:		
	$\frac{dy}{dx} = \frac{-x}{\sqrt{10-x^2}}$ or $\frac{dy}{dx} = -\frac{x}{y}$	**B1**	allow unsimplified
	Result:		
	$y^2 = x^2 + 5y$ after inserted in $y = mx + 5$		
	Attempt to solve with $x^2 + y^2 = 10$	**M1**	Eliminate x or y
	$y = 2, x = \pm\sqrt{6}$	**A1**	both
	$m = \pm\frac{3}{\sqrt{6}}$ oe	**A1**	

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Question 10 **[10]**

10 (i)	tangent touches circle $x^2+(kx-4)^2-2(kx-4)=8$	**M1**	eliminate y or x allow unsimplified
	$k^2x^2+x^2-8kx-2kx+16=0$ or better	**A1**	
	Equal roots as tangent touches circle : $b^2=4ac$	**DM1**	use of discriminant on 3 term quadratic soi
	$(-10k)^2=4(k^2+1)\times 16$	**A1**	
	$36k^2=64$		
	$k=+\frac{4}{3}$ only	**A1**	oe any inequality loses last **A1**
(ii)	$x=\frac{-b}{2a} \rightarrow x=\frac{\frac{4}{3}\times 10}{\frac{25}{9}}$	**M1**	use $x=\frac{-b}{2a}$
	$x=\frac{12}{5}$ $y=-\frac{4}{5}$	**A1A1**	
	OR tangent $y=\frac{4}{3}x-4$ cuts radius	**M1**	find equation of radius and attempt to solve with tangent
	$y=-\frac{3}{4}x+1$		
	at $x=\frac{12}{5}$	**A1**	
	$y=-\frac{4}{5}$	**A1**	
	OR Obtain $25x^2-120x+144=0$ oe	**M1**	obtain any 3 term quadratic using *their* non zero k and reach $x=\ldots$
	$(5x-12)(5x-12)=0$		
	$x=\frac{12}{5} \rightarrow y=-\frac{4}{5}$	**A1A1**	
(iii)	$TP=\sqrt{(0-2.4)^2+(-4+0.8)^2}=4$	**M1A1**	**M1** for using *their* T and $(0,-4)$. Signs must be correct.

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