Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Coordinate geometry of the circle

Time allowed: 1 hour 40 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 1-2, 4-5, 7-8 (Paper 1): no calculator.
Questions 3, 6, 9-10 (Paper 2): calculator allowed.

Total marks

/ 64

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 64 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 The point A has coordinates $(-3, 6)$.
The point B has coordinates $(7, -8)$.

Given that the line AB is the diameter of a circle, find the equation of the circle. [4]

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1	$(x-2)^2+(y+1)^2=74$ oe nfww or $x^2+y^2-4x+2y-69=0$ oe nfww	**4**	**M1** for centre: $\left(\frac{-3+7}{2}, \frac{6-8}{2}\right)$ or $(2,-1)$ soi **M2** for [r =] $\sqrt{74}$ or [r^2 =] 74 or [diameter =] $2\sqrt{74}$ or $c = -69$ or **M1** for $[AB^2 =](-3-7)^2+(6--8)^2$ oe or $[r^2 =](-3-their2)^2+(6-their(-1))^2$ oe or $[r^2 =](7-their2)^2+(-8-their(-1))^2$ oe **Alternative method** **M2** for $\frac{y-6}{x-(-3)}\times\frac{y-(-8)}{x-7}=-1$ or **M1** for e.g. $P(x, y)$ on the circumference means $AP \perp BP \rightarrow m_{AP}\times m_{BP}=-1$ **M1** for $\frac{y^2+2y-48}{x^2-4x-21}=-1$ oe **FT** $their\left(\frac{y-6}{x-(-3)}\times\frac{y-(-8)}{x-7}\right)$ with at most one sign error

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2 Find the coordinates of the points where the line $2y = x - 1$ meets the curve $x^2 + y^2 = 29$. [5]

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2 $(2y+1)^2+y^2=29$ (or $5x^2-2x-115=0$)	M1	M1 for attempt to get an equation in terms of one variable only
leading to $5y^2+4y-28=0$	DM1	DM1 for obtaining a 3 term quadratic equation
(or $x^2+\left(\frac{x-1}{2}\right)^2=29$)	DM1	DM1 for attempt to solve quadratic equation
$x=-\frac{23}{5}, y=-\frac{14}{5}$ and	A1	A1 for a pair of values
$x=5, y=2$	A1	
(5, 2) spotted gets B1	**[5]**	

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3 Solutions to this question by accurate drawing will not be accepted.

A circle has centre (4, 2) and meets the x-axis at $(-2, 0)$.

(a) Find the equation of the circle. [2]

(b) Find, in exact form, the coordinates of the points where the circle meets the y-axis. [3]

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3(a)	$(x-4)^2 + (y-2)^2 = 40$ oe, isw or $x^2 + y^2 - 8x - 4y - 20 = 0$ oe, isw	**B2**	**B1** for $r^2 = 40$ or $r = \sqrt{40}$ oe, soi
3(b)	$16 + (y-2)^2 = 40$ oe	**M1**	**FT** *their* equation of the circle providing it is the equation of a circle
	$(y-2)^2 = 24$ or $y^2 - 4y - 20 = 0$	**A1**	Obtain correct quadratic equation in any form
	$(0, 2 \pm \sqrt{24})$ isw or equivalent exact form	**A1**	

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4 The point A has coordinates $(-2, 4)$.
The point B has coordinates $(6, 10)$.
The point C has coordinates $(12, 2)$.

(a) Find the gradients of the lines AB, AC and BC. [2]

(b) Hence find the equation of the circle which passes through the points A, B and C. [4]

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4(a)	Grad $AB = \frac{6}{8}$ oe Grad $AC = -\frac{2}{14}$ oe Grad $BC = -\frac{8}{6}$ oe	**2**	**B1** for 2 correct unsimplified
4(b)	Centre $(5,\ 3)$	**B1**	**FT** on *their* diameter, allow for $(2,\ 7)$ or $(9,\ 6)$
	Radius^2 = 50 oe soi	**2**	**M1** for use of *their* centre and coordinates of either *A*, *B* or *C* to find the square of the radius or the radius. Centre must not be *A*, *B* or *C*. or Finding the length between any 2 points *A*, *B* or *C* **and** halving it.
	$(x-5)^2+(y-3)^2=50$ oe	**A1**	Allow $\left(5\sqrt{2}\right)^2$ for radius^2

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5 Solutions by accurate drawing will not be accepted.

A circle, C, has equation $(x-5)^2+(y-12)^2=100$.

(a) Find the equation of the tangent to C at the point (11, 4).
Give your answer in the form $ax+by=c$, where a, b and c are integers. [4]

(b) Show that C and the circle with equation $x^2+y^2=4$ do not intersect. [2]

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5(a)	Centre $(5, 12)$ soi	**B1**	
	Gradient of radius = $\frac{12-4}{5-11}$ $\left(=-\frac{4}{3}\right)$	**M1**	
	Equation of normal: $y-4=\frac{3}{4}(x-11)$	**M1**	Must be using *their* perpendicular gradient and the correct point
	$3x-4y=17$	**A1**	
5(b)	Distance between centres = 13 Sum of radii = 12	**M1**	For both
	$13>12$ so no intersection	**A1**	

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6 A circle has equation $x^2 + y^2 - 25 = 0$.
A second circle has the same radius as the first circle, and the coordinates of its centre are both positive.
The two circles intersect at the points A and B.
The line AB has length 6 and is parallel to the line $y = -x$.

Find the equation of the second circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. [5]

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6	Radius: 5 soi	**B1**	Implied by $r^2 = 25$ or 25 used as r^2 in *their* 2nd circle equation
	Centre: $\left(4\sqrt{2}, 4\sqrt{2}\right)$ soi oe	**B2**	**B1** for (5.66, 5.66) or better OR **B1** for distance between centres is 8 soi
	$\left(x-\frac{8}{\sqrt{2}}\right)^2+\left(y-\frac{8}{\sqrt{2}}\right)^2 = 25$ oe or $x^2+y^2-2\left(4\sqrt{2}\right)x-2\left(4\sqrt{2}\right)y+2\left(4\sqrt{2}\right)^2-25=0$ oe	**M1**	**FT** *their*$\left(4\sqrt{2}, 4\sqrt{2}\right)$ in form (k, k) where k is a non-zero constant e.g. $\left(x-their\frac{8}{\sqrt{2}}\right)^2+\left(y-their\frac{8}{\sqrt{2}}\right)^2 = 25$ or $x^2+y^2-2theirx-theiry$ $+2(their)^2-25=0$ oe
	$x^2+y^2-8\sqrt{2}\,x-8\sqrt{2}\,y+39=0$	**A1**	Must be exact

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7 The lines $x = 0$, $x = 4$, $y = 3$ and $y = -1$ are tangents to a circle.

(a) Find the equation of the circle. [3]

The line $y = 2x + a$, where a is a constant, is also a tangent to the circle.

(b) Show that $5x^2 + 4(a-2)x + (a-1)^2 = 0$, and hence find the possible values of a. Give your answers in exact form. [4]

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7(a)	Centre (2, 1)	**B1**	Accept $x = 2$ and $y = 1$
	Radius 2 soi	**B1**	
	$(x-2)^2 + (y-1)^2 = 4$ oe	**B1**	**FT** *their* centre and radius ISW Do not accept 2^2 for 4
7(b)	$(x-2)^2 + (2x+a-1)^2 = 4$	**M1**	Substitute $y = 2x + a$ into *their* circle equation
	$x^2 - 4x + 4 + 4x^2 + 4(a-1)x + (a-1)^2 = 4$	**A1**	Equivalent must be seen as **AG**
	leading to $5x^2 + 4(a-2)x + (a-1)^2 = 0$ **AG**		
	$16(a-2)^2 - 20(a-1)^2 = 0$	**M1**	for finding the discriminant of the given equation and equating to zero
	$a^2 + 6a - 11 = 0$ $a = -3 \pm \sqrt{20}$ or $\frac{-6 \pm \sqrt{80}}{2}$ oe	**A1**	isw from a correct answer isw

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8 A circle with centre C has the equation $x^2 + y^2 - 10x - 4y + 24 = 0$.

(a) Show that the line $y = 2x - 3$ is a tangent to this circle. [3]

(b) Given that this tangent touches the circle at the point P, find the coordinates of P. [2]

(c) Find the equation of the circle which has its centre at P and passes through the origin. [3]

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8(a)	$5x^2-30x+45\ (=0)$ or equivalent 3 term quadratic	**2**	**M1** for attempt to eliminate one variable
	Use of discriminant or solution to show one repeated root or a single solution	**A1**	Must have conclusion e.g. Discriminant = 0 (so tangent) oe One solution only (so tangent)
	Alternative 1		
	Centre of the circle (5, 2) Equation of radius perpendicular to given line $y-2=-\frac{1}{2}(x-5)$ Intersection at $(3,3)$	**(2)**	**M1** for a complete method to obtain the point of intersection
	Show that P lies on the circle or that $CP=\sqrt{5}$	**(A1)**	
	Alternative 2		
	The line $y=kx+n$ touches the circle $(x-p)^2+(y-q)^2=r^2$ if $(-kp+q-n)^2=r^2(1+k^2)$ so $(-(2\times5)+2-3)^2=5(1+2^2)$	**(2)**	**M1** for a complete method showing substitution, allow one slip
	25 = 25, so a tangent	**(A1)**	

8(b)	(3, 3)	**2**	**B1** for each
8(c)	Radius = $3\sqrt{2}$ oe soi	**B1**	
	$(x-3)^2+(y-3)^2$ soi	**B1**	**FT** on *their* answer to **(b)** only
	$(x-3)^2+(y-3)^2=18$ oe	**B1**	ISW any expansion

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9 **(i)** Calculate the coordinates of the points where the line $y = x + 2$ cuts the curve $x^2 + y^2 = 10$. [4]

(ii) Find the exact values of m for which the line $y = mx + 5$ is a tangent to the curve $x^2 + y^2 = 10$. [4]

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9	**(i)**	$(x+2)^2 + x^2 = 10$	**B1**	
		$x^2 + 2x - 3 = 0 \quad \rightarrow \quad (x+3)(x-1) = 0$	**M1**	3 term quadratic with attempt to solve
		Points (1, 3), (–3, –1) isw	**A1** **A1**	both x or a pair both y or second pair
		or elimination of x leads to $y^2 - 2y - 3 = 0$, then as above		
	(ii)	$m^2x^2 + 10mx + 25 + x^2 = 10$	**B1**	
		$(m^2+1)x^2 + 10mx + 15 = 0$		
		$b^2 - 4ac = (0) \quad \rightarrow \quad 100m^2 - 60(m^2+1) = 0$	**M1** **A1**	attempt to use discriminant on three term quadratic. Allow unsimplified
		$m = \pm\sqrt{\frac{3}{2}}$ oe isw	**A1**	cao ± is required
		Alternative solution:		
		$\frac{dy}{dx} = \frac{-x}{\sqrt{10-x^2}}$ or $\frac{dy}{dx} = -\frac{x}{y}$	**B1**	allow unsimplified
		Result:		
		$y^2 = x^2 + 5y$ after inserted in $y = mx + 5$		
		Attempt to solve with $x^2 + y^2 = 10$	**M1**	Eliminate x or y
		$y = 2, x = \pm\sqrt{6}$	**A1**	both
		$m = \pm\frac{3}{\sqrt{6}}$ oe	**A1**	

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10 The line $y = kx - 4$, where k is a positive constant, passes through the point $P\,(0, -4)$ and is a tangent to the curve $x^2 + y^2 - 2y = 8$ at the point T. Find

(i) the value of k, [5]

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(ii) the coordinates of T, [3]

(iii) the length of TP. [2]

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10	**(i)**	tangent touches circle $x^2+(kx-4)^2-2(kx-4)=8$	**M1**	eliminate y or x allow unsimplified
		$k^2x^2+x^2-8kx-2kx+16=0$ or better	**A1**	
		Equal roots as tangent touches circle : $b^2=4ac$	**DM1**	use of discriminant on 3 term quadratic soi
		$(-10k)^2=4(k^2+1)\times 16$	**A1**	
		$36k^2=64$		
		$k=+\frac{4}{3}$ only	**A1**	oe any inequality loses last **A1**
	(ii)	$x=\frac{-b}{2a} \rightarrow x=\frac{\frac{4}{3}\times 10}{\frac{25}{9}}$	**M1**	use $x=\frac{-b}{2a}$
		$x=\frac{12}{5}$ $y=-\frac{4}{5}$	**A1A1**	
		OR tangent $y=\frac{4}{3}x-4$ cuts radius	**M1**	find equation of radius and attempt to solve with tangent
		$y=-\frac{3}{4}x+1$		
		at $x=\frac{12}{5}$	**A1**	
		$y=-\frac{4}{5}$	**A1**	
		OR Obtain $25x^2-120x+144=0$ oe	**M1**	obtain any 3 term quadratic using *their* non zero k and reach $x=\ldots$
		$(5x-12)(5x-12)=0$		
		$x=\frac{12}{5} \rightarrow y=-\frac{4}{5}$	**A1A1**	
	(iii)	$TP=\sqrt{(0-2.4)^2+(-4+0.8)^2}=4$	**M1A1**	**M1** for using *their* T and $(0,-4)$. Signs must be correct.

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