Mark scheme

Differentiation

Additional Mathematics 0606 · Questions from Paper 1 and Paper 2

Total marks

73

MARKS FOR EACH QUESTION

1	2	3	4	5	6	7	8	9	10	Total
4	5	6	7	8	8	9	9	10	7	**73**

Question 1 [4]

1	**(i)** 3 or $\frac{3}{1}$	B1 [1]	
	(ii) $\frac{dy}{dx}=\frac{3\sin t}{4\cos^2 t}$ $\left(=\frac{3\sin t}{3}\right)$	M1	M1 correct substitution in $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$ o.e.
	$=\frac{3\sin\frac{\pi}{6}}{3}$	DM1	
	$=0.5$	A1 [3]	DM1 for use of their '3' and substitution of $\frac{\pi}{6}$.

Question 2 [5]

2	$\frac{d}{dx}(\sin^2 x)=2\sin x\cos x$ soi	**B1**	
	$\cos x\times their(2\sin x\cos x)+$ $(their-\sin x)\times\sin^2 x$	**M1**	**FT** *their* $\frac{d}{dx}(\sin^2 x)$
	$\cos x\times their(2\sin x\cos x)+(-\sin x)\times\sin^2 x$ isw	**A1**	**FT** *their* $\frac{d}{dx}(\sin^2 x)$
	$\frac{\delta y}{h}=their(2\sin 3\cos^2 3-\sin^3 3)$ or better	**M1**	**FT** *their* derivative
	$0.274h$ or $0.2738[08...]h$ where the coefficient of h is rot to 4 or more sf	**A1**	**dep** on correct derivative seen

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Question 3 **[6]**

3(i)	$\dfrac{3x^2\sin 2x - x^3 \times 2\cos 2x}{(\sin 2x)^2}$	**3**	**M1** Quotient rule **A2/1/0** minus one each error isw
3(ii)	$y = \dfrac{\pi^3}{64}$ $\left[= 0.48\ldots\right]$	**B1**	
	$\dfrac{dy}{dx} = \dfrac{3\pi^2}{16}$ [=1.85] oe	**B1**	
	$y = \dfrac{3\pi^2}{16}x - \dfrac{\pi^3}{32}$ $\left[y = 1.85x - 0.97\right]$	**B1**	cao

Question 4 **[7]**

4	Attempts product rule	**M1**	
	$\dfrac{dy}{dx} = 2\cos x - 2x\sin x$ oe	**A1**	
	When $x = \pi$, $\dfrac{dy}{dx} = -2$	**DM1**	**FT** *their* $\left.\dfrac{dy}{dx}\right\|_{x=\pi}$
	Gradient of normal: $\dfrac{1}{2}$	**M1**	**FT** $\dfrac{-1}{\textit{their}\left.\dfrac{dy}{dx}\right\|_{x=\pi}}$
	$y + 2\pi = \dfrac{1}{2}(x - \pi)$ oe	**A1**	**FT** *their* normal gradient
	$(5\pi, 0)$ or $(0, -\dfrac{5}{2}\pi)$ oe, soi	**A1**	
	Area of triangle POQ: $\dfrac{25}{4}\pi^2$ nfww	**A1**	

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Question 5 **[8]**

5(a)	Correct statement for area e.g. $\frac{1}{2}r^2\alpha = 9$ or $\frac{1}{2}rs = 9$	**B1**	
	Finds an equation that can be used to eliminate α or s e.g. $\alpha = \frac{18}{r^2}$ or $r\alpha = \frac{18}{r}$ or $s = \frac{18}{r}$	**M1**	
	Correct substitution and completion to given answer e.g. $P = 2r + r \times \frac{18}{r^2} = 2r + \frac{18}{r}$	**A1**	
5(b)	Correct derivative: $2 - \frac{18}{r^2}$ oe isw	**B1**	
	$2 - \frac{18}{r^2} = 0$ and solves for r	**M1**	**FT** *their* $\frac{\mathrm{d}P}{\mathrm{d}r} = 0$ providing of the form $2 + \frac{n}{r^k}$ where n and k are non-zero integers
	$r = 3$ only soi	**A1**	
	$P = 12$ only soi	**A1**	
	[2nd derivative =] $\frac{36}{r^3}$ oe **and** which is positive for $r = 3$ →minimum **or** as $r > 0$ [2nd derivative =] > 0 →minimum **or** [2nd derivative =] $\frac{36}{3^3}$ oe →minimum OR correctly finds the values of the first derivative at $3 \pm h$ where h is small → minimum	**A1**	**dep** on $r = 3$ and no other value

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Question 6 **[8]**

6	$\frac{dy}{dx} = 3x^2 + 3x - 2$	**M1**	Allow one error in the coefficient of x^2or x
	Gradient of tangent at $A = -2$ nfww	**A1**	
	Gradient of normal at $A = \frac{1}{2}$	**M1**	**Dep** for an attempt $-1 \div$ *their* gradient – must come from differentiation
	Equation of normal is $y - 1 = \frac{1}{2}x$ oe	**M1**	**Dep** for the normal equation using *their* normal gradient **and** $(0, 1)$
	$1+\frac{1}{2}x = x^3 + \frac{3}{2}x^2 - 2x + 1$	**M1**	**Dep** For solving simultaneously curve with normal **FT** *their* normal equation
	$x(2x^2 + 3x - 5) = 0$ oe nfww	**A1**	
	$[x](x - 1)(2x + 5) = 0$	**M1**	For an attempt solve *their* 3-term quadratic or trying to solve *their* cubic using long division
	$x = 1$ and $x = -\frac{5}{2}$ nfww	**A1**	

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Question 7 **[9]**

7(i)	$\ln(2x+1)-\ln(2x-1)$	**B1**	
7(ii)	attempt to differentiate	**M1**	
	$\frac{dy}{dx}=\frac{2}{2x+1}-\frac{2}{2x-1}+4$	**A1**	all correct
	attempt to obtain in required form	**DM1**	
	$=\frac{16x^2-8}{4x^2-1}$	**A1**	A1 all correct
7(iii)	When $\frac{dy}{dx}=0$, $16x^2-8=0$	**M1**	setting $\frac{dy}{dx}=0$ and attempt to solve
	$x=\frac{1}{\sqrt{2}}$ only	**A1**	
7(iv)	$\frac{d^2y}{dx^2}=\frac{32x(4x^2-1)-8x(16x^2-8)}{(4x^2-1)^2}$	**M1**	attempt at second derivative and conclusion or equivalent method
	When $x=\frac{1}{\sqrt{2}}$ $\frac{d^2y}{dx^2}$ is + ve, so minimum	**A1**	

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Question 8 **[9]**

8(a)	$\frac{dy}{dx}=\frac{\left(\left(x+5\right)\times\frac{3}{2}\times2\left(2x+1\right)^{\frac{1}{2}}-\left(2x+1\right)^{\frac{3}{2}}\right)}{\left(x+5\right)^{2}}$	**3**	**B1** for $\frac{3}{2}\times2\left(2x+1\right)^{\frac{1}{2}}$ oe **M1** for attempt to differentiate a quotient or product **A1** for all terms other than $\frac{3}{2}\times2\left(2x+1\right)^{\frac{1}{2}}$ correct
	$\frac{\left(2x+1\right)^{\frac{1}{2}}}{\left(x+5\right)^{2}}\left(x+14\right)$	**A1**	
8(b)	When $\frac{dy}{dx}=0$, $x=-14$ and $x=-0.5$ but $x\geqslant0$ so no stationary point	**B1**	FT on *their* $\left(x+14\right)$
8(c)	$\frac{5\sqrt{3}p}{12}$, $\frac{15\sqrt{3}p}{36}$ or $0.722p$	**2**	**M1** for calculation of $\frac{dy}{dx}$ when $x=1$ and multiplication by p
8(d)	$\frac{25\sqrt{3}}{24}$ or 1.80 oe	**2**	**M1** for multiplication of *their* $\frac{dy}{dx}$ calculated in **(c)** by 2.5, must be numeric

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Question 9 [10]

9 (i)	$AB = 2r\sin\theta$ or $\sqrt{r^2 + r^2 - 2r^2\cos 2\theta}$ or $\dfrac{r\sin 2\theta}{\sin\left(\dfrac{\pi}{2} - \theta\right)}$ or $\dfrac{r\sin 2\theta}{\cos\theta}$	**B1**	
(ii)	$2r\sin\theta + 2r\theta = 20$ $r = \dfrac{10}{\theta + \sin\theta}$	**M1** **A1**	for use of (i) + arc length = 20, oe must be convinced
(iii)	$\dfrac{\mathrm{d}r}{\mathrm{d}\theta} = -\dfrac{10(1+\cos\theta)}{(\theta+\sin\theta)^2}$ When $\theta = \dfrac{\pi}{6}$, $\dfrac{\mathrm{d}r}{\mathrm{d}\theta} = -17.8$	**M1** **A2,1,0** **A1**	for a correct attempt to differentiate –1 each error allow awrt -17.8
(iv)	$\dfrac{\mathrm{d}r}{\mathrm{d}t} = 15$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{\mathrm{d}r}{\mathrm{d}t} \div \dfrac{\mathrm{d}r}{\mathrm{d}\theta}$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -0.842$	**B1** **M1** **A1**	may be implied for use of $\dfrac{15}{\textit{their}\ \text{(iii)}}$ allow -0.84 or -0.843

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Question 10 **[7]**

10(a)	$\frac{PQ}{8}=\frac{12-h}{12}$ oe or $\left(\frac{12-h}{12}\right)^2=\frac{\text{Area }\Delta PQR}{16\sqrt{3}}$ oe	**M1**	
	$PQ=\frac{8(12-h)}{12}$ oe or Area $\Delta PQR=16\sqrt{3}\left(\frac{12-h}{12}\right)^2$ oe	**A1**	
	$V=\frac{1}{2}\times\left(\frac{8(12-h)}{12}\right)^2\times\sin\frac{\pi}{3}\times h$ oe or $16\sqrt{3}\left(\frac{12-h}{12}\right)^2\times h$	**M1**	**FT** *their PQ* or Area ΔPQR providing of correct structure
	$V=\frac{\sqrt{3}}{9}(h^3-24h^2+144h)$	**A1**	
10(b)	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\sqrt{3}}{9}(3h^2-48h+144)$	**B1**	**FT** *their* $V=\frac{\sqrt{3}}{9}(h^3-24h^2+144h)$ if of the same structure
	their $\frac{\sqrt{3}}{9}(3h^2-48h+144)=0$ and factorises/solves	**M1**	must be a 3-term quadratic; must be an attempt at a derivative
	4 oe identified as the only solution; cao	**A1**	

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