Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Differentiation

Time allowed: 1 hour 50 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 1, 6-9 (Paper 1): no calculator.
Questions 2-5, 10 (Paper 2): calculator allowed.

Total marks

/ 73

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 73 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 The rate of change of a variable x with respect to time t is $4\cos^2 t$.

(i) Find the rate of change of x with respect to t when $t = \frac{\pi}{6}$. [1]

The rate of change of a variable y with respect to time t is $3\sin t$.

(ii) Using your result from part **(i)**, find the rate of change of y with respect to x when $t = \frac{\pi}{6}$. [3]

Connected rates of change and small increments Click for more variations

1	**(i)** 3 or $\frac{3}{1}$	B1 [1]	
	(ii) $\frac{dy}{dx} = \frac{3\sin t}{4\cos^2 t}$ $\left(= \frac{3\sin t}{3}\right)$	M1	M1 correct substitution in $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ o.e.
	$= \frac{3\sin\frac{\pi}{6}}{3}$	DM1	
	$= 0.5$	A1 [3]	DM1 for use of their '3' and substitution of $\frac{\pi}{6}$.

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2 Variables x and y are such that $y = \cos x \sin^2 x$. Use differentiation to find the approximate change in y as x increases from 3 to $3 + h$, where h is small. [5]

Connected rates of change and small increments Click for more variations

2	$\frac{\mathrm{d}}{\mathrm{d}x}(\sin^2 x) = 2\sin x \cos x$ soi	**B1**	
	$\cos x \times their(2\sin x \cos x) +$ $(their - \sin x) \times \sin^2 x$	**M1**	**FT** *their* $\frac{\mathrm{d}}{\mathrm{d}x}(\sin^2 x)$
	$\cos x \times their(2\sin x \cos x) + (-\sin x) \times \sin^2 x$ isw	**A1**	**FT** *their* $\frac{\mathrm{d}}{\mathrm{d}x}(\sin^2 x)$
	$\frac{\delta y}{h} = their(2\sin 3\cos^2 3 - \sin^3 3)$ or better	**M1**	**FT** *their* derivative
	$0.274h$ or $0.2738[08...]h$ where the coefficient of h is rot to 4 or more sf	**A1**	**dep** on correct derivative seen

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3 A curve has equation $y = \frac{x^3}{\sin 2x}$. Find

(i) $\frac{\mathrm{d}y}{\mathrm{d}x}$, [3]

(ii) the equation of the tangent to the curve at the point where $x = \frac{\pi}{4}$. [3]

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Differentiation, tangents, normals and stationary points Click for more variations

3(i)	$\dfrac{3x^2\sin 2x - x^3 \times 2\cos 2x}{(\sin 2x)^2}$	**3**	**M1** Quotient rule **A2/1/0** minus one each error isw
3(ii)	$y = \dfrac{\pi^3}{64}$ $[= 0.48\ldots]$	**B1**	
	$\dfrac{dy}{dx} = \dfrac{3\pi^2}{16}$ [=1.85] oe	**B1**	
	$y = \dfrac{3\pi^2}{16}x - \dfrac{\pi^3}{32}$ $[y = 1.85x - 0.97]$	**B1**	cao

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4 A curve has equation $y = 2x\cos x$. The normal to the curve at $(\pi, -2\pi)$ meets the x-axis and y-axis at points P and Q. Find the exact area of triangle POQ. [7]

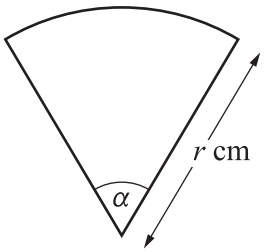
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Differentiation, tangents, normals and stationary points Click for more variations

4	Attempts product rule	**M1**	
	$\frac{dy}{dx} = 2\cos x - 2x\sin x$ oe	**A1**	
	When $x = \pi$, $\frac{dy}{dx} = -2$	**DM1**	**FT** *their* $\left.\frac{dy}{dx}\right\|_{x=\pi}$
	Gradient of normal: $\frac{1}{2}$	**M1**	**FT** $\frac{-1}{\textit{their}\left.\frac{dy}{dx}\right\|_{x=\pi}}$
	$y + 2\pi = \frac{1}{2}(x - \pi)$ oe	**A1**	**FT** *their* normal gradient
	$(5\pi, 0)$ or $(0, -\frac{5}{2}\pi)$ oe, soi	**A1**	
	Area of triangle POQ: $\frac{25}{4}\pi^2$ nfww	**A1**	

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5 The diagram shows a design for a logo. The logo is a sector of a circle, radius r cm, with angle α radians.



The area of the logo is $9\,\text{cm}^2$.

(a) Show that the perimeter, P cm, of the logo is given by

$$P = 2r + \frac{18}{r}.$$ [3]

(b) Given that r can vary, find the stationary value of P and determine its nature. [5]

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Maxima and minima problems Click for more variations

5(a)	Correct statement for area e.g. $\frac{1}{2}r^2\alpha = 9$ or $\frac{1}{2}rs = 9$	**B1**	
	Finds an equation that can be used to eliminate α or s e.g. $\alpha = \frac{18}{r^2}$ or $r\alpha = \frac{18}{r}$ or $s = \frac{18}{r}$	**M1**	
	Correct substitution and completion to given answer e.g. $P = 2r + r \times \frac{18}{r^2} = 2r + \frac{18}{r}$	**A1**	
5(b)	Correct derivative: $2 - \frac{18}{r^2}$ oe isw	**B1**	
	$2 - \frac{18}{r^2} = 0$ and solves for r	**M1**	**FT** *their* $\frac{\mathrm{d}P}{\mathrm{d}r} = 0$ providing of the form $2 + \frac{n}{r^k}$ where n and k are non-zero integers
	$r = 3$ only soi	**A1**	
	$P = 12$ only soi	**A1**	
	[2nd derivative =] $\frac{36}{r^3}$ oe **and** which is positive for $r = 3$ →minimum **or** as $r > 0$ [2nd derivative =] > 0 →minimum **or** [2nd derivative =] $\frac{36}{3^3}$ oe →minimum OR correctly finds the values of the first derivative at $3 \pm h$ where h is small → minimum	**A1**	**dep** on $r = 3$ and no other value

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6 The normal to the curve $y = x^3 + \frac{3}{2}x^2 - 2x + 1$ at the point where $x = 0$ cuts the curve again at two other points.

Find the x-coordinates of these two points. [8]

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Differentiation, tangents, normals and stationary points Click for more variations

6	$\frac{dy}{dx} = 3x^2 + 3x - 2$	**M1**	Allow one error in the coefficient of x^2or x
	Gradient of tangent at $A = -2$ nfww	**A1**	
	Gradient of normal at $A = \frac{1}{2}$	**M1**	**Dep** for an attempt $-1 \div$ *their* gradient – must come from differentiation
	Equation of normal is $y - 1 = \frac{1}{2}x$ oe	**M1**	**Dep** for the normal equation using *their* normal gradient **and** $(0, 1)$
	$1+\frac{1}{2}x = x^3 + \frac{3}{2}x^2 - 2x + 1$	**M1**	**Dep** For solving simultaneously curve with normal **FT** *their* normal equation
	$x(2x^2 + 3x - 5) = 0$ oe nfww	**A1**	
	$[x](x - 1)(2x + 5) = 0$	**M1**	For an attempt solve *their* 3-term quadratic or trying to solve *their* cubic using long division
	$x = 1$ and $x = -\frac{5}{2}$ nfww	**A1**	

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7 **(i)** Write $\ln\left(\frac{2x+1}{2x-1}\right)$ as the difference of two logarithms. [1]

A curve has equation $y = \ln\left(\frac{2x+1}{2x-1}\right) + 4x$ for $x > \frac{1}{2}$.

(ii) Using your answer to part **(i)** show that $\frac{dy}{dx} = \frac{ax^2+b}{4x^2-1}$, where a and b are integers. [4]

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(iii) Hence find the x-coordinate of the stationary point on the curve. [2]

(iv) Determine the nature of this stationary point. [2]

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Differentiation, tangents, normals and stationary points Click for more variations

7(i)	$\ln(2x+1)-\ln(2x-1)$	**B1**	
7(ii)	attempt to differentiate	**M1**	
	$\frac{dy}{dx}=\frac{2}{2x+1}-\frac{2}{2x-1}+4$	**A1**	all correct
	attempt to obtain in required form	**DM1**	
	$=\frac{16x^2-8}{4x^2-1}$	**A1**	A1 all correct
7(iii)	When $\frac{dy}{dx}=0$, $16x^2-8=0$	**M1**	setting $\frac{dy}{dx}=0$ and attempt to solve
	$x=\frac{1}{\sqrt{2}}$ only	**A1**	
7(iv)	$\frac{d^2y}{dx^2}=\frac{32x(4x^2-1)-8x(16x^2-8)}{(4x^2-1)^2}$	**M1**	attempt at second derivative and conclusion or equivalent method
	When $x=\frac{1}{\sqrt{2}}$ $\frac{d^2y}{dx^2}$ is + ve, so minimum	**A1**	

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8 A curve has equation $y = \dfrac{(2x+1)^{\frac{3}{2}}}{x+5}$ for $x \geqslant 0$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(2x+1)^{\frac{1}{2}}}{(x+5)^2}(Ax+B)$, where A and B are integers to be found. [4]

(b) Show that there are no stationary points on this curve. [1]

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(c) Find the approximate change in y when x increases from 1 to $1+p$, where p is small. [2]

(d) Given that when $x = 1$ the rate of change in x is 2.5 units per second, find the corresponding rate of change in y. [2]

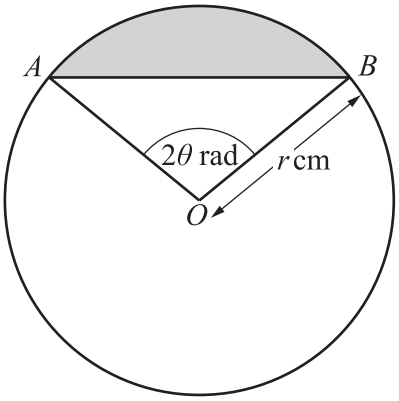
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Differentiation, tangents, normals and stationary points Click for more variations

8(a)	$\frac{dy}{dx}=\frac{\left((x+5)\times\frac{3}{2}\times2(2x+1)^{\frac{1}{2}}-(2x+1)^{\frac{3}{2}}\right)}{(x+5)^2}$	**3**	**B1** for $\frac{3}{2}\times2(2x+1)^{\frac{1}{2}}$ oe **M1** for attempt to differentiate a quotient or product **A1** for all terms other than $\frac{3}{2}\times2(2x+1)^{\frac{1}{2}}$ correct
	$\frac{(2x+1)^{\frac{1}{2}}}{(x+5)^2}(x+14)$	**A1**	
8(b)	When $\frac{dy}{dx}=0$, $x=-14$ and $x=-0.5$ but $x\geqslant0$ so no stationary point	**B1**	FT on *their* $(x+14)$
8(c)	$\frac{5\sqrt{3}p}{12}$, $\frac{15\sqrt{3}p}{36}$ or $0.722p$	**2**	**M1** for calculation of $\frac{dy}{dx}$ when $x=1$ and multiplication by p
8(d)	$\frac{25\sqrt{3}}{24}$ or 1.80 oe	**2**	**M1** for multiplication of *their* $\frac{dy}{dx}$ calculated in **(c)** by 2.5, must be numeric

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9



The diagram shows a circle, centre O, radius r cm. The points A and B lie on the circle such that angle $AOB = 2\theta$ radians.

(i) Find, in terms of r and θ, an expression for the length of the chord AB. [1]

(ii) Given that the perimeter of the shaded region is 20 cm, show that $r = \dfrac{10}{\theta + \sin\theta}$. [2]

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(iii) Given that r and θ can vary, find the value of $\frac{\mathrm{d}r}{\mathrm{d}\theta}$ when $\theta = \frac{\pi}{6}$. [4]

(iv) Given that r is increasing at the rate of $15\,\mathrm{cm\,s^{-1}}$, find the corresponding rate of change of θ when $\theta = \frac{\pi}{6}$. [3]

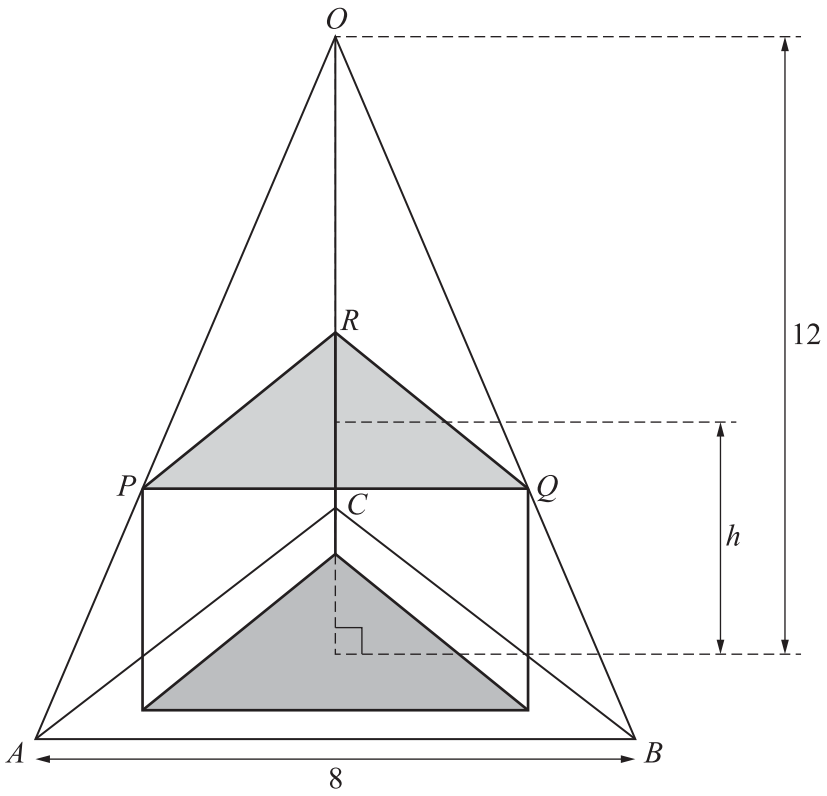
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Differentiation, tangents, normals and stationary points Click for more variations

9 **(i)**	$AB = 2r\sin\theta$ or $\sqrt{r^2 + r^2 - 2r^2\cos 2\theta}$ or $\dfrac{r\sin 2\theta}{\sin\left(\dfrac{\pi}{2} - \theta\right)}$ or $\dfrac{r\sin 2\theta}{\cos\theta}$	**B1**	
(ii)	$2r\sin\theta + 2r\theta = 20$ $r = \dfrac{10}{\theta + \sin\theta}$	**M1** **A1**	for use of (i) + arc length = 20, oe must be convinced
(iii)	$\dfrac{\mathrm{d}r}{\mathrm{d}\theta} = -\dfrac{10(1+\cos\theta)}{(\theta+\sin\theta)^2}$ When $\theta = \dfrac{\pi}{6}$, $\dfrac{\mathrm{d}r}{\mathrm{d}\theta} = -17.8$	**M1** **A2,1,0** **A1**	for a correct attempt to differentiate −1 each error allow awrt −17.8
(iv)	$\dfrac{\mathrm{d}r}{\mathrm{d}t} = 15$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{\mathrm{d}r}{\mathrm{d}t} \div \dfrac{\mathrm{d}r}{\mathrm{d}\theta}$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -0.842$	**B1** **M1** **A1**	may be implied for use of $\dfrac{15}{\textit{their}\ \text{(iii)}}$ allow −0.84 or −0.843

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10 In this question all lengths are in centimetres.



The diagram shows a right triangular prism of height h inside a right pyramid.
The pyramid has a height of 12 and a base that is an equilateral triangle, ABC, of side 8.
The base of the prism sits on the base of the pyramid.
Points P, Q and R lie on the edges OA, OB and OC, respectively, of the pyramid $OABC$.
Pyramids $OABC$ and $OPQR$ are similar.

(a) Show that the volume, V, of the triangular prism is given by $V = \frac{\sqrt{3}}{9}(ah^3 + bh^2 + ch)$ where a, b and c are integers to be found. [4]

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(b) It is given that, as h varies, V has a maximum value. Find the value of h that gives this maximum value of V. [3]

Maxima and minima problems Click for more variations

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10(a)	$\frac{PQ}{8}=\frac{12-h}{12}$ oe or $\left(\frac{12-h}{12}\right)^2=\frac{\text{Area }\Delta PQR}{16\sqrt{3}}$ oe	**M1**	
	$PQ=\frac{8(12-h)}{12}$ oe or Area $\Delta PQR=16\sqrt{3}\left(\frac{12-h}{12}\right)^2$ oe	**A1**	
	$V=\frac{1}{2}\times\left(\frac{8(12-h)}{12}\right)^2\times\sin\frac{\pi}{3}\times h$ oe or $16\sqrt{3}\left(\frac{12-h}{12}\right)^2\times h$	**M1**	**FT** *their PQ* or Area ΔPQR providing of correct structure
	$V=\frac{\sqrt{3}}{9}(h^3-24h^2+144h)$	**A1**	
10(b)	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\sqrt{3}}{9}(3h^2-48h+144)$	**B1**	**FT** *their* $V=\frac{\sqrt{3}}{9}(h^3-24h^2+144h)$ if of the same structure
	their $\frac{\sqrt{3}}{9}(3h^2-48h+144)=0$ and factorises/solves	**M1**	must be a 3-term quadratic; must be an attempt at a derivative
	4 oe identified as the only solution; cao	**A1**	

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