Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Factors and polynomials

Time allowed: 1 hour 45 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 3-7, 10 (Paper 1): no calculator.
Questions 1-2, 8-9 (Paper 2): calculator allowed.

Total marks

/ 70

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 70 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 It is given that $\mathrm{p}(x) = ax^3 - 7x^2 - bx + 9$, where a and b are constants.
$x - 3$ is a factor of $\mathrm{p}(x)$.
When $\mathrm{p}(x)$ is divided by $x + 2$ the remainder is -35.

Find the values of a and b. [5]

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Reduced to 98% to fit the page (wider than A4).

1	$27a - 63 - 3b + 9 = 0$ oe	**B1**	
	$-8a - 28 + 2b + 9 = -35$ oe	**B1**	
	Eliminates one unknown and solves for a or b	**M1**	**FT** *their* equations provided both equations are linear with terms in a and b
	$a = 2$, $b = 0$ nfww	**A2**	**A1** for each

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2 DO NOT USE A CALCULATOR IN THIS QUESTION.

The polynomial p is defined by $p(x) = ax^3 - 3x^2 - 3x + b$, where a and b are constants.

(a) Given that $x = 2$ and $x = -1$ are roots of the equation $p(x) = 0$, find a and b. [3]

(b) Solve the equation $p(x) = 0$. [2]

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2(a)	$8a - 12 - 6 + b = 0$ oe and $-a - 3 + 3 + b = 0$ oe and $a = 2$, $b = 2$	**B3**	**B1** for $8a - 12 - 6 + b = 0$ oe **B1** for $-a - 3 + 3 + b = 0$ oe
2(b)	$[2x^3 - 3x^2 - 3x + 2 =]$ $(x - 2)$, $(x + 1)$, $(2x - 1)$ or $(x^2 - x - 2)$, $(2x - 1)$ **and** $x = 2, \frac{1}{2}, -1$ OR $[2x^3 - 3x^2 - 3x + 2 =]$ $(x + 1)$, $(2x^2 - 5x + 2)$ or $(x - 2)$, $(2x^2 + x - 1)$ **and** correct factorisation or method of solution of the quadratic **and** $x = 2, \frac{1}{2}, -1$ OR $2 - 1 + x = -\left(\frac{-3}{2}\right)$ or $2 - 1 + x = \frac{3}{2}$ or for $2 \times -1 \times x = \frac{-2}{2}$ or $2 \times -1 \times x = -1$ **and** $x = 2, \frac{1}{2}, -1$	**B2**	**B1** for $[2x^3 - 3x^2 - 3x + 2 =]$ $(x - 2)$ and $(x + 1)$ seen or $(x^2 - x - 2)$ seen or $(x + 1)$, $(2x^2 - 5x + 2)$ or $(x - 2)$, $(2x^2 + x - 1)$ **B1** for $2 - 1 + x = -\left(\frac{-3}{2}\right)$ or $\frac{3}{2}$ or for $2 \times -1 \times x = \frac{-2}{2}$ or -1

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3 The polynomial p is such that $p(x) = x^3 + Ax + 30$, where A is a constant.
When $p(x)$ is divided by $x + 2$ the remainder is 84.

Write $p(x)$ as a product of linear factors. [5]

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3	$A = -31$	**B1**	
	$x - 1$ or $x - 5$ or $x + 6$	**B1**	For finding a factor
	$[x-1](x^2+x-30)$ $[x-5](x^2+5x-6)$ $[x+6](x^2-6x+5)$	**2**	**M1** for a quadratic factor with 2 correct terms or for one further linear factor
	$(x - 1)(x - 5)(x + 6)$	**A1**	

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4 The function f is such that $f(x) = 4x^3 - 8x^2 + ax + b$, where a and b are constants. It is given that $2x - 1$ is a factor of $f(x)$ and that when $f(x)$ is divided by $x + 2$ the remainder is 20. Find the remainder when $f(x)$ is divided by $x - 1$. [6]

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4	$f(-2): -2a + b = 84$	M1 A1	M1 for substitution of a correct value of x
	$f\left(\frac{1}{2}\right): \frac{1}{2}a + b = \frac{3}{2}$	A1	A1 for each correct equation (allow unsimplified)
	$a = -33,\ b = 18$	M1, A1	M1 for solution to obtain a and b
	$f(1) = -19$	√B1 [6]	√B1 on their a and b

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5 DO NOT USE A CALCULATOR IN THIS QUESTION.

The polynomial p is such that $\mathrm{p}(x) = 6x^3 - 35x^2 + 34x + 45$.

(a) Find $\mathrm{p}(x)$ in the form $(2x-5)\mathrm{q}(x) + r$, where $\mathrm{q}(x)$ is a polynomial and r is a constant. [3]

(b) Hence write the expression $\mathrm{p}(x) - 5$ as a product of linear factors. [2]

(c) Hence write down the solutions of the equation $\mathrm{p}(x) = 5$. [1]

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5(a)	$q(x) = 3x^2 - 10x - 8$ $r = 5$	**3**	**M1** for a valid attempt to obtain the quotient by algebraic long division, synthetic division, factorising $(2x - 5)$ or by forming an identity **A1** for each
5(b)	$[p(x) - 5 =](3x^2 - 10x - 8)(2x - 5)$	**M1**	For a correct attempt to factorise
	$(3x + 2)(x - 4)(2x - 5)$	**A1**	Must see $p(x) - 5$ as a product of linear factors
5(c)	$x = -\frac{2}{3},\ 4,\ \frac{5}{2}$	**B1**	**FT** on *their* 3 terms quadratic $q(x)$

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6 The polynomial p is such that $\mathrm{p}(x) = ax^3 + 11x^2 + bx + c$, where a, b and c are integers.
It is given that $\mathrm{p}'(0) = 12$.
It is also given that $x + 3$ is a factor of p.
When p is divided by $x - 1$ the remainder is 16.

Find the values of a, b and c. [6]

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6	$b=12$	**2**	**M1** for attempt at differentiation
	$-27a+99-3b+c=0$ $a+11+b+c=16$	**2**	**M1** for attempt at $p(-3)=0$ or $p(1)=16$
	$a=2$ $c=-9$	**2**	**M1** for attempt to solve *their* equations

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7 The polynomial P is given by $\mathrm{P}(x) = ax^3 + bx^2 + 3x + 2$, where a and b are integers. $\mathrm{P}(x)$ has a factor of $2x + 1$. $\mathrm{P}(x)$ has a remainder of -6 when divided by $x + 1$.

(a) Find the values of a and b. [5]

(b) Show that the equation $\mathrm{P}(x) = 0$ has only one real root. [3]

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7(a)	$P\left(-\frac{1}{2}\right): -\frac{a}{8}+\frac{b}{4}-\frac{3}{2}+2=0$	**M1**	Allow one arithmetic error. **Must** be equated to 0 soi Allow unsimplified
	$P(-1): -a+b-3+2=-6$	**M1**	Allow unsimplified **Must** be equated to -6 soi
	$-a+2b+4=0$ oe $-a+b+5=0$ oe	**A1**	For both allow unsimplified
	$a=6,\ \ b=1$	**2**	**M1 dep** on at least one previous M1 for attempt to solve *their* simultaneous equations to find at least one of *their* unknowns. **A1** for both.
7(b)	$(2x+1)(3x^2-x+2)$	**2**	**M1** for correct attempt to obtain 2 terms of the quadratic for *their* P(x). Must divide by $(2x + 1)$ **A1** for correct quadratic $(3x^2-x+2)$
	For $3x^2-x+2$, the dicriminant is <0 so only one real root of $-\frac{1}{2}$ oe	**B1**	Must have a valid attempt to evaluate the discriminant.

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8 The cubic equation $x^3 + ax^2 + bx - 36 = 0$ has a repeated positive integer root.

(i) If the repeated root is $x = 3$ find the other positive root and the value of a and of b. [4]

(ii) There are other possible values of a and b for which the cubic equation has a repeated positive integer root. In each case state all three integer roots of the equation. [4]

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8(i)	Other root $= 4$	**B1**	
	$\mathrm{f}(x) = (x-3)(x-3)(x-4)$ $= x^3 - 10x^2 + 33x - 36$	**M1**	multiply out $(x-3)(x-3)(x \pm p)$
	$a = -10$ $\quad b = 33$	**A2**	**A1** for each Can be implied by correct cubic
8(ii)	$x = 6,\ x = 6,\ x = 1$ $x = 2,\ x = 2,\ x = 9$ $x = 1,\ x = 1,\ x = 36$	**B4**	**B1** for each of first two sets **B2** for third set

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9 Do not use a calculator in this question.

The line $y = 4x - 6$ intersects the curve $y = 10x^3 - 19x^2 - x$ at the points A, B, and C. Given that C is the point (2, 2), find the coordinates of the midpoint of AB. [10]

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9	Eliminates y: $10x^3 - 19x^2 - x = 4x - 6$ oe	**M1**	
	Collects terms: $10x^3 - 19x^2 - 5x + 6 = 0$	**A1**	
	$x - 2$ is a factor soi	**B1**	
	Correct quadratic factor: $(10x^2 + x - 3)$	**B2**	**B1** for any two correct elements of quadratic factor
	Factorises *their* 3-term quadratic or solves *their* 3-term quadratic = 0: $(5x + 3)(2x - 1)$ oe	**M1**	
	$(-0.6, -8.4), (0.5, -4)$	**A2**	**A1** for $x = -0.6$, $x = 0.5$ or $y = -8.4$, $y = -4$ nfww or for one correct (x, y) pair
	$\left(\frac{their(-0.6+0.5)}{2}, \frac{their(-8.4+-4)}{2}\right)$ oe	**M1**	
	$(-0.05, -6.2)$ oe	**A1**	

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10 A function f is such that $f(x) = 4x^3 + 4x^2 + ax + b$. It is given that $2x - 1$ is a factor of both $f(x)$ and $f'(x)$.

(i) Show that $b = 2$ and find the value of a. [5]

Using the values of a and b from part **(i)**,

(ii) find the remainder when $f(x)$ is divided by $x + 3$, [2]

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(iii) express $f(x)$ in the form $f(x) = (2x - 1)(px^2 + qx + r)$, where p, q and r are integers to be found, [2]

(iv) find the values of x for which $f(x) = 0$. [2]

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10	**(i)** $f\left(\frac{1}{2}\right): \frac{3}{2} + \frac{a}{2} + b = 0$	M1	M1 for use of $x = \frac{1}{2}$ and equating to zero
	$f'(x) = 12x^2 + 8x + a$	M1	M1 for differentiation
	$f'\left(\frac{1}{2}\right): 3 + 4 + a = 0$	M1	M1 for attempt to obtain $a = -7$ from $f'\left(\frac{1}{2}\right)$
	Leading to $a = -7$	A1	
	and $b = 2$	A1	
		[5]	
	(ii) $f(-3) = -49$	M1	M1 for use of $x = -3$ in either the remainder theorem or algebraic long division.
		A1	
		[2]	
	(iii) $f(x) = (2x - 1)(2x^2 + 3x - 2)$	M1, A1	M1 for attempt to obtain quadratic factor
		[2]	
	(iv) $f(x) = (2x - 1)(2x - 1)(x + 2)$	B1	B1 for each – must be correct from work
	Leading to $x = 0.5, -2$	B1	
		[2]	

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