Mark scheme

Functions

Additional Mathematics 0606 · Questions from Paper 1 and Paper 2

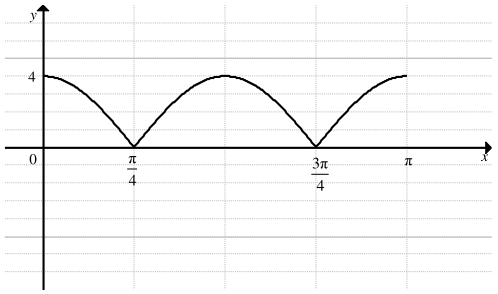
Total marks

64

MARKS FOR EACH QUESTION

1	2	3	4	5	6	7	8	9	10	Total
3	4	5	5	6	7	7	8	9	10	**64**

Question 1 **[3]**

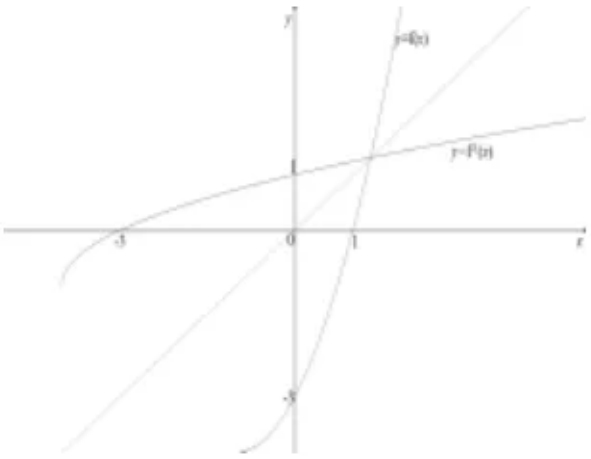
1	Correct sketch  	**3**	**B1** for correct shape, with 3 consistent maxima, 2 cusps on the x-axes and reasonable symmetry **B1** for $\left(\frac{\pi}{4},\ 0\right)$ and $\left(\frac{3\pi}{4},\ 0\right)$ either seen on the graph or stated; must have attempted a graph of correct shape **B1** for starts with (0, 4) **and** ends with (π, 4) and (0, 4) either seen on the graph or stated; must have attempted a graph correct shape

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Question 2 **[4]**

2	$\frac{3\sqrt{x+2}}{\sqrt{x+2}+4}=1$ soi OR $g(x)=f^{-1}(1)$ and $f^{-1}(x)=\frac{4x}{3-x}$ oe	**2**	**M1** for $f(\sqrt{x+2})\ [=1]$ soi or for $g(x)=f^{-1}(1)$ soi
	$x=2$ nfww	**2**	**M1 FT** for correct equation without an algebraic fraction e.g. $\sqrt{x+2}=2$ **FT** *their* $\frac{3\sqrt{x+2}}{\sqrt{x+2}+4}=1$ providing **M1** previously awarded and fg(x) is an algebraic fraction
	Alternative method		
	Solves $\frac{3x}{x+4}=1$ to find $x = 2$ and states $g(x)=2$	**(2)**	**M1** for solving $f(x)=1$ to find $x = k$ and states $g(x)=k$, where k is a constant
	$x=2$ nfww	**(2)**	**M1 FT** for $\sqrt{x+2}=k$ **FT** $g(x)=k$ *their* providing **M1** previously awarded

Question 3 **[5]**

3(a)	It is a one-one function because of the given restricted domain or because $x \geqslant -1$	**B1**	
3(b)		**4**	**B1** for $y=f(x)$ for $x>-1$ only **B1** for 1 on x-axis and –3 on y-axis for $y=f(x)$ **B1** for $y=f^{-1}(x)$ as a reflection of $y=f(x)$ in the line $y=x$, maybe implied by intercepts with axes **B1** for 1 on y-axis and –3 on x-axis for $y=f^{-1}(x)$

Question 4 **[5]**

4(a)	Least value of a when $2+a=1$ so $a=-1$.	**2**	**B1** for range of f: $2+a$, may be implied by $a+1$ or $\sqrt{a+1}$
4(b)	$\sqrt{\left(2e^{x}+5\right)-1}$ $[=3]$	**M1**	For correct order
	$x=0.916$	**2**	**M1** for correct attempt to solve to obtain x = …

Question 5 **[6]**

5(a)	Correct V shape with vertex on positive x-axis	**B1**	
	(0, 7)	**B1**	
	$\left(\frac{7}{5},\ 0\right)$	**B1**	

5(b)	$x=2$	**B1**	
	$5x-7=their(-3)$ oe, soi or $25x-35=their(-15)$ oe, soi	**M1**	
	$x=\frac{4}{5}$ oe	**A1**	
	Alternative method		
	$25x^{2}-70x+40=0$ oe	**(B1**	
	factorising e.g. $(5x-4)(x-2)$	**M1**	
	$x=2,\ \frac{4}{5}$	**A1)**	

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Question 6 [7]

6(a)(i)	Valid explanation e.g. The line $x = k$, where $-10 \leqslant k \leqslant 10$ cuts the curve in one point only	**B1**	
6(a)(ii)	$\dfrac{\frac{x}{x-1}}{\frac{x}{x-1}-1}$	**M1**	
	Correct simplification to x	**A1**	
6(a)(iii)	Valid explanation e.g. f and f^{-1} are the same or f is self-inverse oe	**B1**	
6(a)(iv)	Valid explanation e.g. The curve is symmetrical about line $y = x$	**B1**	
6(b)	$1 < g \leqslant 2$	**B1**	
6(c)	$x \neq -\frac{1}{3}$	**B1**	

Question 7 [7]

7(a)	$2(e^x + 1)^2 - 1$ [= 8]	**M1**	
	$e^x = -1+\sqrt{\frac{9}{2}}$ oe	**A1**	
	$x = \ln\left(\frac{3}{\sqrt{2}} - 1\right)$ isw or 0.115 or 0.1145[06…] rot to 4 or more dp	**A1**	
7(b)	f is not one-one, hence f^{-1} does not exist oe	**B1**	
	$g^{-1}(x) = \ln(x - 1)$	**2**	**M1** for $x = \ln(y - 1)$ and a swop of variables at some point or $y = \ln(x + 1)$ or $e^x = y - 1$ and $y = \ln x - 1$
	$x > 1$	**B1**	

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Question 8 **[8]**

8(a)(i)	Range f^{-1}: $0.5 \leqslant f^{-1} \leqslant 1.5$	**B1**	
	Domain f^{-1}: $0 \leqslant x \leqslant \frac{2\sqrt{2}}{3}$ oe	**B2**	**B1** for 0 and $\frac{2\sqrt{2}}{3}$ in an incorrect inequality or for $x \geqslant 0$ or $x \leqslant \frac{2\sqrt{2}}{3}$
8(a)(ii)	Correctly collects terms ready to factorise e.g. $4x^2 - 4x^2y^2 = 1$ or $4y^2x^2 - 4y^2 = -1$ or simplifies to subject in one term only e.g. $\frac{1}{4y^2} = 1 - x^2$ or $-\frac{1}{4x^2} = y^2 - 1$ oe	**M1**	
	Correctly factorises and/or rearranges at least as far as: $x^2 = \frac{1}{4-4y^2}$ or $y^2 = \frac{-1}{4x^2-4}$ oe	**M1**	**FT** only if of equivalent difficulty
	$\left[f^{-1}(x) =\right] \sqrt{\frac{1}{4-4x^2}}$ or $\left[y =\right] \sqrt{\frac{-1}{4x^2-4}}$ oe, isw	**A1**	
8(b)	Correct order of composition: $gf(x) = e^{\left(\frac{\sqrt{4x^2-1}}{2x}\right)^2}$	**M1**	
	$gf(x) = e^{\left(1-\frac{1}{4x^2}\right)}$ isw	**A1**	

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Question 9 **[9]**

9(a)	$f > 9$	**B1**	Allow y but not x
9(b)	It is a one-one function because of the restricted domain	**B1**	
9(c)	$x = (2y+3)^2$ or equivalent	**M1**	For a correct attempt to find the inverse
	$y = \dfrac{\sqrt{x}-3}{2}$	**M1**	For correct rearrangement
	$f^{-1} = \dfrac{\sqrt{x}-3}{2}$	**A1**	Must have correct notation
9(d)	$x > 9$	**B1**	**FT** on *their* (a)
9(e)	$f(\ln(x+4)) = 49$	**M1**	For correct order
	$(2\ln(x+4)+3)^2 = 49$ $\ln(x+4) = 2$	**M1**	For correct attempt to solve, dep on previous M mark, as far as $x =$
	$x = e^2 - 4$	**A1**	

Question 10 **[10]**

10(a)	Many-many (mapping) oe or A vertical line meets graph more than once (or twice) oe	**B1**	
10(b)	<table><tr><td></td><td>✓</td><td></td></tr><tr><td>✓</td><td></td><td>✓</td></tr><tr><td></td><td>✓</td><td></td></tr><tr><td>✓</td><td></td><td></td></tr></table>	**4**	**B1** for each correct row
10(c)(i)	$\sin x \geqslant \frac{1}{2}$	**M1**	May be implied. Allow >
	$a = 150$ seen	**A1**	
10(c)(ii)	$0 \leqslant gf(x) \leqslant \frac{1}{\sqrt{2}}$	**2**	**B1** for each correct value with correct sign associated with it
10(c)(iii)	No. Range of g $\not\subset$ domain of g oe	**B1**	

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