Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Functions

Time allowed: 1 hour 40 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 3, 9-10 (Paper 1): no calculator.
Questions 1-2, 4-8 (Paper 2): calculator allowed.

Total marks

/ 64

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 64 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

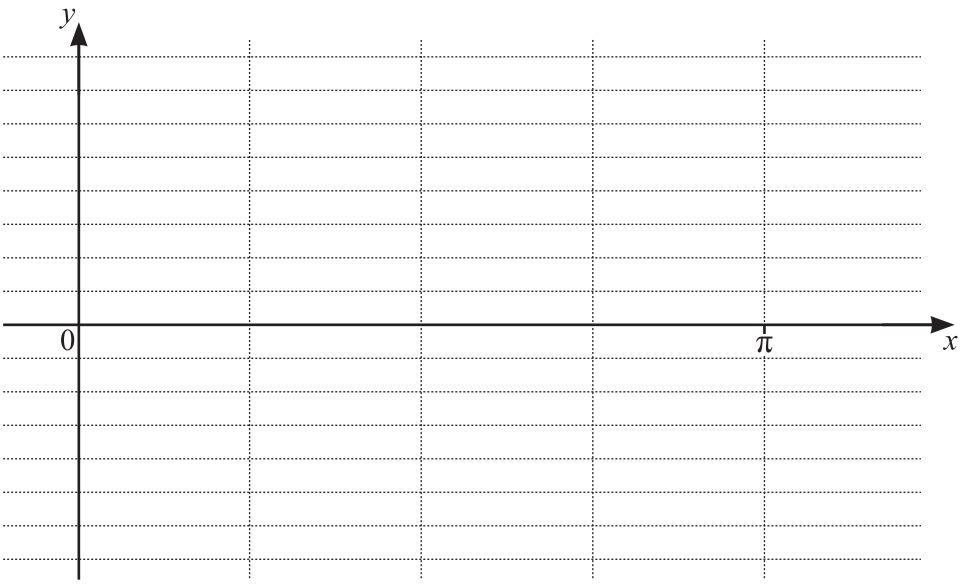
Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

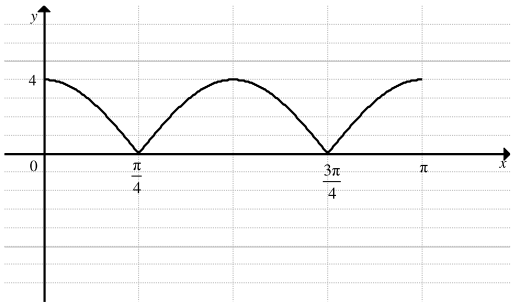
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Print at 100% (Actual size) so diagrams keep their real measurements.

1 On the axes below, sketch the graph of $y = |4\cos 2x|$ for $0 \leqslant x \leqslant \pi$, giving the coordinates of any points where the graph meets the axes. [3]



Modulus graphs Click for more variations

1	Correct sketch  	**3**	**B1** for correct shape, with 3 consistent maxima, 2 cusps on the x-axes and reasonable symmetry **B1** for $\left(\frac{\pi}{4}, 0\right)$ and $\left(\frac{3\pi}{4}, 0\right)$ either seen on the graph or stated; must have attempted a graph of correct shape **B1** for starts with (0, 4) **and** ends with (π, 4) and (0, 4) either seen on the graph or stated; must have attempted a graph correct shape

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2 Functions f and g are such that

$$\mathrm{f}(x) = \frac{3x}{x+4} \quad \text{for } x > 0$$
$$\mathrm{g}(x) = \sqrt{x+2} \quad \text{for } x > -2\,.$$

Solve the equation $\mathrm{fg}(x) = 1$. [4]

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Domain, range, inverse and composite functions Click for more variations

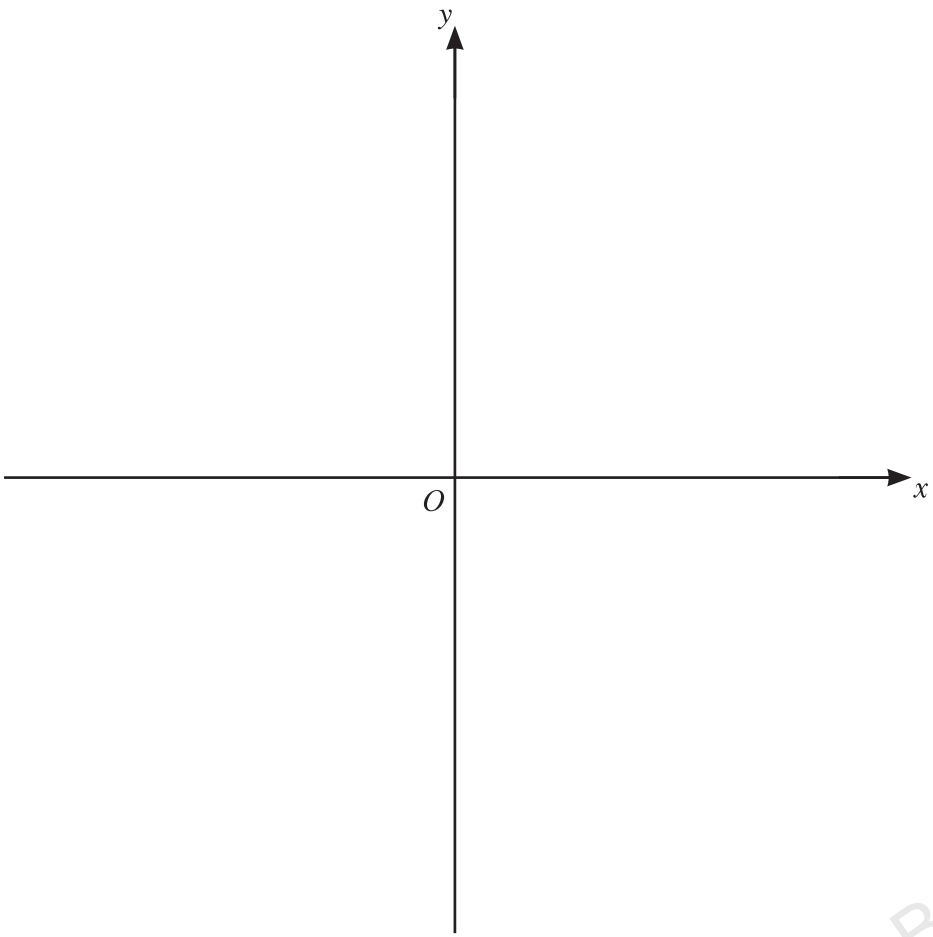
2	$\frac{3\sqrt{x+2}}{\sqrt{x+2}+4}=1$ soi OR $g(x)=f^{-1}(1)$ and $f^{-1}(x)=\frac{4x}{3-x}$ oe	**2**	**M1** for $f(\sqrt{x+2})$ $[=1]$ soi or for $g(x)=f^{-1}(1)$ soi
	$x=2$ nfww	**2**	**M1 FT** for correct equation without an algebraic fraction e.g. $\sqrt{x+2}=2$ **FT** *their* $\frac{3\sqrt{x+2}}{\sqrt{x+2}+4}=1$ providing **M1** previously awarded and fg(x) is an algebraic fraction
	Alternative method		
	Solves $\frac{3x}{x+4}=1$ to find $x = 2$ and states $g(x)=2$	**(2)**	**M1** for solving $f(x)=1$ to find $x = k$ and states $g(x)=k$, where k is a constant
	$x=2$ nfww	**(2)**	**M1 FT** for $\sqrt{x+2}=k$ **FT** $g(x)=k$ *their* providing **M1** previously awarded

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3 $\mathrm{f}(x) = x^2 + 2x - 3 \quad \text{for } x \geqslant -1$

(a) Given that the minimum value of $x^2 + 2x - 3$ occurs when $x = -1$, explain why $\mathrm{f}(x)$ has an inverse. [1]

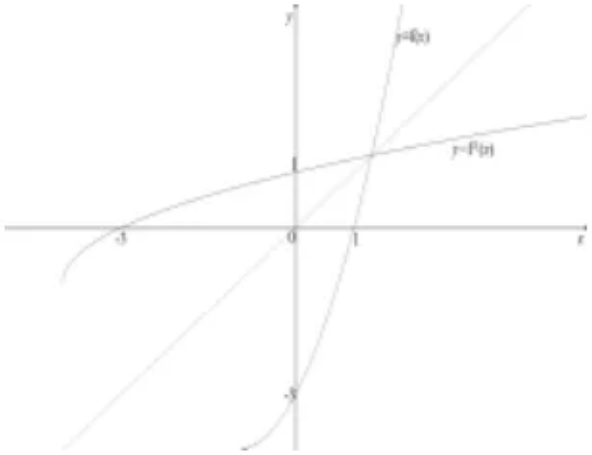
(b) On the axes below, sketch the graph of $y = \mathrm{f}(x)$ and the graph of $y = \mathrm{f}^{-1}(x)$. Label each graph and state the intercepts on the coordinate axes.



[4]

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Domain, range, inverse and composite functions Click for more variations

3(a)	It is a one-one function because of the given restricted domain or because $x \geqslant -1$	**B1**	
3(b)		**4**	**B1** for $y = \mathrm{f}(x)$ for $x > -1$ only **B1** for 1 on x-axis and -3 on y-axis for $y = \mathrm{f}(x)$ **B1** for $y = \mathrm{f}^{-1}(x)$ as a reflection of $y = \mathrm{f}(x)$ in the line $y = x$, maybe implied by intercepts with axes **B1** for 1 on y-axis and -3 on x-axis for $y = \mathrm{f}^{-1}(x)$

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4 It is given that $f(x) = 2e^x + a$ for $x \geqslant 0$, where a is an integer

and $g(x) = \sqrt{x-1}$ for $x \geqslant 1$.

(a) Find the least value of a so that the function gf exists for all $x \geqslant 0$. [2]

(b) In the case where $a = 5$, solve the equation $gf(x) = 3$.
Give your answer correct to 3 decimal places. [3]

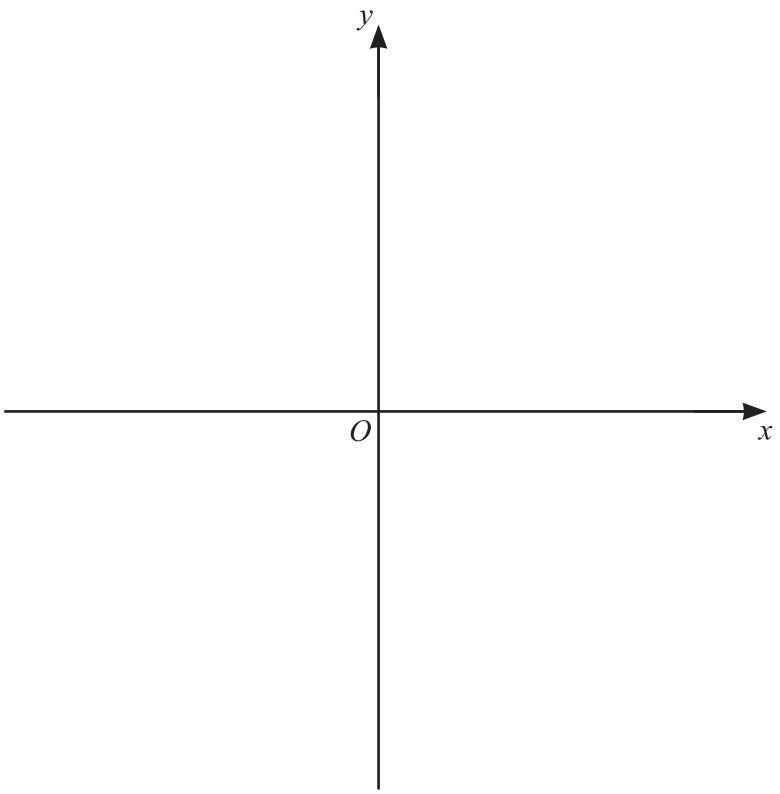
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Domain, range, inverse and composite functions Click for more variations

4(a)	Least value of a when $2+a=1$ so $a=-1$.	**2**	**B1** for range of f: $2+a$, may be implied by $a+1$ or $\sqrt{a+1}$
4(b)	$\sqrt{\left(2e^{x}+5\right)-1}$ $[=3]$	**M1**	For correct order
	$x=0.916$	**2**	**M1** for correct attempt to solve to obtain $x = \ldots$

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5 **(a)** On the axes below, sketch the graph of $y = |5x - 7|$, showing the coordinates of the points where the graph meets the coordinate axes. [3]



(b) Solve $5|5x - 7| - 1 = 14$. [3]

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Modulus graphs Click for more variations

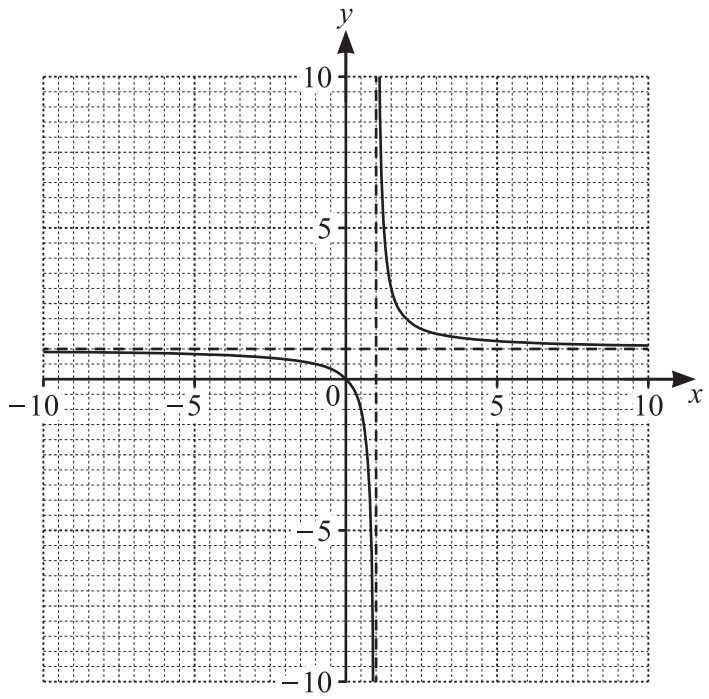
5(a)	Correct V shape with vertex on positive x-axis	**B1**	
	(0, 7)	**B1**	
	$\left(\frac{7}{5}, 0\right)$	**B1**	

5(b)	$x = 2$	**B1**	
	$5x - 7 = their(-3)$ oe, soi or $25x - 35 = their(-15)$ oe, soi	**M1**	
	$x = \frac{4}{5}$ oe	**A1**	
	Alternative method		
	$25x^2 - 70x + 40 = 0$ oe	**(B1**	
	factorising e.g. $(5x - 4)(x - 2)$	**M1**	
	$x = 2, \frac{4}{5}$	**A1)**	

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6 **(a)** $f(x) = \dfrac{x}{x-1}$ for $-10 \leqslant x \leqslant 10, x \neq 1$.

The diagram shows the graph of $y = f(x)$.



(i) Use the diagram to explain why f is a function. [1]

(ii) Find $ff(x)$, giving your answer in its simplest form. [2]

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(iii) Using your answer to **part (ii)** state the relationship between the functions f and f^{-1}. [1]

(iv) Explain how the diagram shows the relationship between f and f^{-1}. [1]

(b) A function g is defined by $g(x) = \frac{x}{x-1}$ for $x \geqslant 2$. Find the range of g. [1]

(c) A function h is defined by $h(x) = \frac{2x}{3x+1}$ for the largest possible domain. State the domain of h. [1]

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Domain, range, inverse and composite functions Click for more variations

6(a)(i)	Valid explanation e.g. The line $x = k$, where $-10 \leqslant k \leqslant 10$ cuts the curve in one point only	**B1**	
6(a)(ii)	$\dfrac{\frac{x}{x-1}}{\frac{x}{x-1}-1}$	**M1**	
	Correct simplification to x	**A1**	

6(a)(iii)	Valid explanation e.g. f and f^{-1} are the same or f is self-inverse oe	**B1**	
6(a)(iv)	Valid explanation e.g. The curve is symmetrical about line $y = x$	**B1**	
6(b)	$1 < g \leqslant 2$	**B1**	
6(c)	$x \neq -\frac{1}{3}$	**B1**	

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7 The functions f and g are defined as follows, for all real values of x.

$$\mathrm{f}(x) = 2x^2 - 1$$
$$\mathrm{g}(x) = \mathrm{e}^x + 1$$

(a) Solve the equation $\mathrm{fg}(x) = 8$. [3]

(b) For each of the functions f and g, either explain why the inverse function does not exist or find the inverse function, stating its domain. [4]

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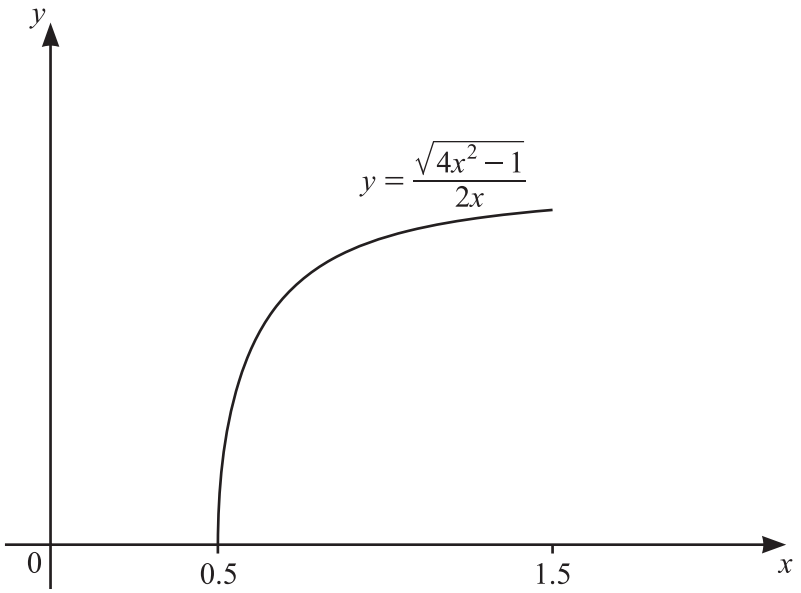
Domain, range, inverse and composite functions Click for more variations

7(a)	$2(e^x + 1)^2 - 1$ [= 8]	**M1**	
	$e^x = -1+\sqrt{\frac{9}{2}}$ oe	**A1**	
	$x = \ln\left(\frac{3}{\sqrt{2}} - 1\right)$ isw or 0.115 or 0.1145[06…] rot to 4 or more dp	**A1**	
7(b)	f is not one-one, hence f^{-1} does not exist oe	**B1**	
	$g^{-1}(x) = \ln(x - 1)$	**2**	**M1** for $x = \ln(y - 1)$ and a swop of variables at some point or $y = \ln(x + 1)$ or $e^x = y - 1$ and $y = \ln x - 1$
	$x > 1$	**B1**	

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8 The function f is defined by $\mathrm{f}(x) = \dfrac{\sqrt{4x^2 - 1}}{2x}$ for $0.5 \leqslant x \leqslant 1.5$.

The diagram shows a sketch of $y = \mathrm{f}(x)$.



(a) (i) It is given that f^{-1} exists. Find the domain and range of f^{-1}. [3]

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(ii) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

(b) The function g is defined by $\mathrm{g}(x) = \mathrm{e}^{x^2}$ for all real x. Show that $\mathrm{gf}(x) = \mathrm{e}^{\left(1-\frac{a}{bx^2}\right)}$, where a and b are integers. [2]

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Domain, range, inverse and composite functions Click for more variations

8(a)(i)	Range f^{-1}: $0.5 \leqslant f^{-1} \leqslant 1.5$	**B1**	
	Domain f^{-1}: $0 \leqslant x \leqslant \frac{2\sqrt{2}}{3}$ oe	**B2**	**B1** for 0 and $\frac{2\sqrt{2}}{3}$ in an incorrect inequality or for $x \geqslant 0$ or $x \leqslant \frac{2\sqrt{2}}{3}$

8(a)(ii)	Correctly collects terms ready to factorise e.g. $4x^2 - 4x^2y^2 = 1$ or $4y^2x^2 - 4y^2 = -1$ or simplifies to subject in one term only e.g. $\frac{1}{4y^2} = 1 - x^2$ or $-\frac{1}{4x^2} = y^2 - 1$ oe	**M1**	
	Correctly factorises and/or rearranges at least as far as: $x^2 = \frac{1}{4 - 4y^2}$ or $y^2 = \frac{-1}{4x^2 - 4}$ oe	**M1**	**FT** only if of equivalent difficulty
	$\left[f^{-1}(x) =\right] \sqrt{\frac{1}{4 - 4x^2}}$ or $\left[y =\right] \sqrt{\frac{-1}{4x^2 - 4}}$ oe, isw	**A1**	
8(b)	Correct order of composition: $gf(x) = e^{\left(\frac{\sqrt{4x^2-1}}{2x}\right)^2}$	**M1**	
	$gf(x) = e^{\left(1 - \frac{1}{4x^2}\right)}$ isw	**A1**	

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9 $\mathrm{f}: x \mapsto (2x+3)^2 \quad \text{for } x > 0$

(a) Find the range of f. [1]

(b) Explain why f has an inverse. [1]

(c) Find f^{-1}. [3]

(d) State the domain of f^{-1}. [1]

(e) Given that $\mathrm{g}: x \mapsto \ln(x+4) \quad \text{for } x > 0$, find the exact solution of $\mathrm{fg}(x) = 49$. [3]

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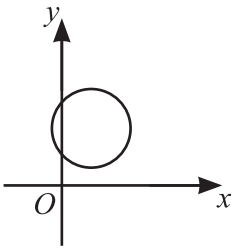
Domain, range, inverse and composite functions Click for more variations

9(a)	$f > 9$	**B1**	Allow y but not x
9(b)	It is a one-one function because of the restricted domain	**B1**	

9(c)	$x = (2y+3)^2$ or equivalent	**M1**	For a correct attempt to find the inverse
	$y = \dfrac{\sqrt{x}-3}{2}$	**M1**	For correct rearrangement
	$f^{-1} = \dfrac{\sqrt{x}-3}{2}$	**A1**	Must have correct notation
9(d)	$x > 9$	**B1**	**FT** on *their* (a)
9(e)	$f(\ln(x+4)) = 49$	**M1**	For correct order
	$(2\ln(x+4)+3)^2 = 49$ $\ln(x+4) = 2$	**M1**	For correct attempt to solve, dep on previous M mark, as far as $x =$
	$x = e^2 - 4$	**A1**	

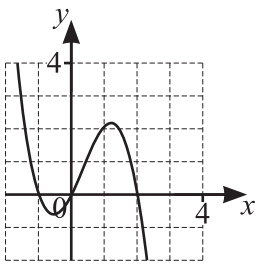
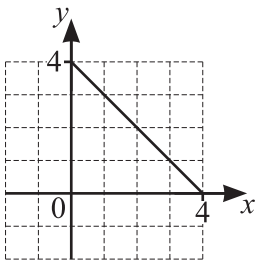
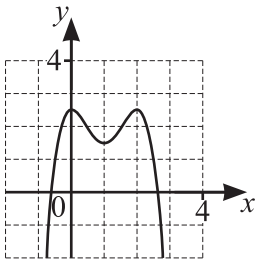
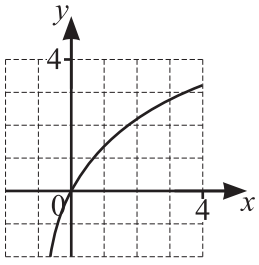
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10 (a)



Explain why this graph does **not** represent a function. [1]

(b) The table shows the graphs of four different functions.

	is one-one	is many-one	is its own inverse
			
			
			
			

Tick (✓) each correct box in the table.
There may be more than one tick in a row or a column. [4]

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(c) Functions f and g are defined as follows.

$\text{f}: x \mapsto \sin x \qquad \text{for } 30° \leqslant x \leqslant a°$

$\text{g}: x \mapsto \sqrt{x - \frac{1}{2}} \qquad \text{for } x \geqslant \frac{1}{2}$

It is given that the function gf exists.

(i) Find the value of a so that the domain of gf is as large as possible.

You may use the information that $\sin 30° = \frac{1}{2}$. [2]

(ii) For the domain found in **part (i)**, find the range of the function gf. [2]

(iii) Determine whether the function g^2 exists. [1]

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Domain, range, inverse and composite functions Click for more variations

10(a)	Many-many (mapping) oe or A vertical line meets graph more than once (or twice) oe	**B1**	
10(b)	<table><tr><td></td><td>✓</td><td></td></tr><tr><td>✓</td><td></td><td>✓</td></tr><tr><td></td><td>✓</td><td></td></tr><tr><td>✓</td><td></td><td></td></tr></table>	**4**	**B1** for each correct row
10(c)(i)	$\sin x \geqslant \frac{1}{2}$	**M1**	May be implied. Allow >
	$a = 150$ seen	**A1**	
10(c)(ii)	$0 \leqslant \text{gf}(x) \leqslant \frac{1}{\sqrt{2}}$	**2**	**B1** for each correct value with correct sign associated with it
10(c)(iii)	No. Range of g $\not\subset$ domain of g oe	**B1**	

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