Mark scheme

Integration

Additional Mathematics 0606 · Questions from Paper 1 and Paper 2

Total marks

73

MARKS FOR EACH QUESTION

1	2	3	4	5	6	7	8	9	10	Total
6	6	7	7	9	7	7	7	8	9	**73**

Question 1 **[6]**

1(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{5}{3x+2}\right)=-5(3x+2)^{-2}\times 3$	**M1**	$-5(3x+2)^{-2}$
		A1	$\times 3$
1(ii)	$\int\frac{30}{(3x+2)^2}\,\mathrm{d}x=\left[\frac{-10}{(3x+2)}\right]$	**M1**	$\frac{1}{(3x+2)}$
		A1	$\times -10$
1(iii)	$\left[\frac{-10}{(3x+2)}\right]_1^2=-\frac{10}{8}+\frac{10}{5}$	**M1**	Insert limits and subtract
	$=\frac{3}{4}$	**A1**	

Question 2 **[6]**

2	$\frac{(x+1)^2}{x^2}=\frac{x^2+2x+1}{x^2}=1+\frac{2}{x}+\frac{1}{x^2}$	**2**	**B1** for expanding numerator seen anywhere. **M1** for attempt to divide *their* three term numerator by x^2.
	$\int 1+\frac{2}{x}+\frac{1}{x^2}\,\mathrm{d}x=x+2\ln x-\frac{1}{x}+(c)$	**2**	**A2/1/0** minus 1 each error or omission.
	$\left[4-2\ln 4-\frac{1}{4}\right]-\left[2+2\ln 2-\frac{1}{2}\right]$	**M1**	**Dep** insert 4 and 2 into *their* three or two term integrand and subtract correctly.
	$=\frac{9}{4}+2\ln 2$	**A1**	oe must be exact two terms. isw

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Question 3 [7]

3(a)(i)	$\frac{1}{5}e^{5x-2}(+c)$	**2**	**B1** for ke^{5x-2} where k is a non-zero constant
3(a)(ii)	$-\frac{1}{3}\ln(4-3x)(+c)$	**2**	**B1** for $-\frac{1}{3}\ln 4-3x$ or $kln(4-3x)$ where k is a non-zero constant
3(b)	$2\tan\left(\frac{1}{2}x\right)$	**M2**	**M1** for $k\tan\left(\frac{1}{2}x\right)$ where k is a non-zero constant
	Fully justified completion to given answer e.g. $2\tan\frac{\pi}{4}-2\tan\frac{\pi}{6}=2\left(1-\frac{\sqrt{3}}{3}\right)$	**A1**	Or $2(\tan\frac{\pi}{4}-\tan\frac{\pi}{6})=2\left(1-\frac{\sqrt{3}}{3}\right)$

Question 4 [7]

4(a)	$\frac{dy}{dx}=2\times3(3x+1)\ln(3x+1)+\frac{3(3x+1)^2}{3x+1}$ Simplified to: $\frac{dy}{dx}=3(3x+1)(1+2\ln(3x+1))$	**3**	**B1** for $\frac{3}{3x+1}$ **M1** for attempt to differentiate a product **A1** for all terms other than $\frac{3}{3x+1}$ correct. Allow unsimplified.
4(b)	$k(3x+1)^2\ln(3x+1)$ soi	**B1**	
	$\int(3x+1)dx=\frac{3x^2}{2}+x\ (+c)$	**B1**	May be a multiple May be seen as $\frac{1}{3}\times\frac{1}{2}\times(3x+1)^2$
	$\frac{1}{6}(3x+1)^2\ln(3x+1)-\frac{3x^2}{4}-\frac{x}{2}+c$	**B2**	**B1** for two correct algebraic terms For **B2** must have $(+c)$

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Question 5 **[9]**

5 (i)	$[y =]\dfrac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}}(+c)$ oe	**B2**	or **B1** for $(2x+1)^{\frac{1}{2}+1}$
	$10=\dfrac{2}{6}(2(4)+1)^{\frac{3}{2}}+c$ oe	**M1**	for valid attempt to find c; condone slips e.g. omission of power or sign error
	$y=\dfrac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}}+c$ seen and $c = 1$ or	**A1**	must have $y = \ldots$; condone f(x) = ...
	$y=\dfrac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}}+1$ isw		
(ii)	$\int\left(\dfrac{1}{3}(2x+1)^{\frac{3}{2}}+1\right)dx=\dfrac{1}{15}(2x+1)^{\frac{5}{2}}+x(+const)$	**B1 + B1**	**B1** for $(2x+1)^{\frac{3}{2}+1}$, **B1** for $\dfrac{1}{15}(2x+1)^{\frac{5}{2}}$
		B1ft	**B1 ft** *their c* from **(i)** provided $c \neq 0$
	$\left[\dfrac{1}{15}(2x+1)^{\frac{5}{2}}+x\right]_0^{1.5}=$		
	$\left[\dfrac{1}{15}(2(1.5)+1)^{\frac{5}{2}}+(1.5)\right]-\left[\dfrac{1}{15}(2(0)+1)^{\frac{5}{2}}+0\right]$	**M1**	for a genuine attempt to find F(1.5) – F(0) in an attempt to integrate *their y*; if their F(0) is 0 must see at least their F(1.5)– 0; condone + c as long as their c is **not** numerical.
	$\dfrac{107}{30}$ oe isw	**A1**	if decimal 3.57 or more accurate e.g. 3.566

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Question 6 **[7]**

6	$\int \cos\left(4x-\frac{\pi}{4}\right)\mathrm{d}x = \frac{1}{4}\sin\left(4x-\frac{\pi}{4}\right)(+c)$	**B2**	**B1** for $k\sin\left(4x-\frac{\pi}{4}\right)$ where $k>0$ or $k=-\frac{1}{4}$
	$\frac{3}{4}=\frac{1}{4}\sin\left(4\left(\frac{3\pi}{16}\right)-\frac{\pi}{4}\right)+c$	**M1**	**FT** *their k* providing **B1** awarded
	$-\frac{1}{16}\cos\left(4x-\frac{\pi}{4}\right)+\frac{1}{2}x \quad (+A)$	**2**	**M1 FT** for $m\cos\left(4x-\frac{\pi}{4}\right)+\left(their\,\frac{1}{2}\right)x \quad (+A)$ **FT** $their\,k\sin\left(4x-\frac{\pi}{4}\right)+their\,c$ providing at least **B1 M1** awarded
	$y=-\frac{1}{16}\cos\left(4x-\frac{\pi}{4}\right)+\frac{1}{2}x+\frac{5\pi}{32}$ oe, cao	**2**	**M1 FT** for $\frac{\pi}{4}=-\frac{1}{16}\cos\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)+\frac{1}{2}\left(\frac{3\pi}{16}\right)+A$ **FT** $(their\,m)\cos\left(4x-\frac{\pi}{4}\right)+\left(their\,\frac{1}{2}\right)x+A$ providing previous **M1** awarded

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Question 7 **[7]**

7(a)	$[x=]\ \frac{\pi}{24}, \frac{5\pi}{24}$ and no extras in range	**2**	**B1** for $[x=]\ \frac{\pi}{24}$ or $\frac{5\pi}{24}$ ignoring extras or for $4x = \frac{\pi}{6}$ **and** $\frac{5\pi}{6}$ soi
7(b)	Correct and complete plan soi e.g. $\int_{their\frac{\pi}{24}}^{their\frac{5\pi}{24}}\left(\sin(4x)-\frac{1}{2}\right)\mathrm{d}x$ or $2\int_{their\frac{\pi}{24}}^{\frac{\pi}{8}}\left(\sin(4x)-\frac{1}{2}\right)\mathrm{d}x$	**M1**	**FT** *their* limits in radians from part (a) or $\int_{their\frac{\pi}{24}}^{their\frac{5\pi}{24}}\sin 4x\,\mathrm{d}x - \frac{1}{2}\left(their\frac{5\pi}{24} - their\frac{\pi}{24}\right)$
	Integrates sin 4x: $-\frac{1}{4}\cos 4x$	**B2**	**B1** for $k\cos 4x$ where $k < 0$ or $k = \frac{1}{4}$
	Substitutes exact limits in correct order: $-\frac{1}{4}\cos 4\left(\frac{5\pi}{24}\right)-\left[-\frac{1}{4}\cos 4\left(\frac{\pi}{24}\right)\right]$ oe soi	**M1**	**FT** *their* exact limits and *their* $-\frac{1}{4}\cos 4x$ providing at least **B1** awarded
	$\frac{\sqrt{3}}{4}-\frac{\pi}{12}$ or exact equivalent	**A1**	

Question 8 **[7]**

8	$-x^4 + 3x^2 + 4\ (= 0)$	**M1**	For attempt to solve simultaneously Allow one term error
	$(x^2 - 4)(x^2 + 1) = 0$ oe	**M1**	**Dep** for an attempt to solve *their* 3-term quadratic equation in terms of x^2
	$x = 2$ and -2 only	**A1**	
	$\int(-x^4+3x^2+4)\mathrm{d}x$	**M1**	For an attempt to integrate *their* quadratic–quartic or the other way around with one term correctly integrated Allow without limits
	$\left[-\frac{x^5}{5}+x^3+4x\right]_{-2}^{2}$	**A1**	All correct with correct limits of 2 and -2 or with limits of 0 and 2 or -2 and 0 then double the area of the integral
	$-\frac{32}{5} + 8 + 8 - (\frac{32}{5} - 8 - 8)$	**M1**	**Dep** for correct use of *their* limits into *their* integral substituted correctly If *their* integration – must see substitution of limits to award the mark
	$= \frac{96}{5}$ or 19.2	**A1**	

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Question 9 **[8]**

9(a)	$\frac{5}{2}$, 1	**B1**	

9(b)	Area of trapezium = $\frac{7}{4}$	**B1**	**FT** on *their y* coordinates
	Area under curve = $\int_1^2 \left(\frac{3}{x+1} - 1 + \frac{2}{x}\right) \mathrm{d}x$	**B1**	Condone omission of limits here
	$3\ln(x+1) - x + 2\ln x$	**2**	**M1** for at least one correct log term
	$(3\ln 3 - 2 - 2\ln 2) - (3\ln 2 - 1)$	**M1**	**Dep** on previous **M** mark
	$\ln\frac{27}{2} - 1$	**M1**	**Dep** on previous **M** mark for simplification using log rules correctly
	$\frac{11}{4} - \ln\left(\frac{27}{2}\right)$	**A1**	

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Question 10 **[9]**

10(a)	$A\left(-\frac{5}{2},\ 0\right)$	**B1**	
	$x(-5-2x)+3=0$ $2x^2+5x-3=0$ $(2x-1)(x+3)=0$	**M1**	For attempt to eliminate one variable, obtain a 3-term quadratic equation = 0 and attempt to solve
	$B\left(\frac{1}{2},\ -6\right)$	**A1**	Allow A1 if just the x-coordinates or just the y-coordinates are given
10(b)	Area of triangle $=\frac{1}{2}\left(\frac{5}{2}+\frac{1}{2}\right)\times 6\ ,=9$	**M1**	For attempt at triangle using *their* values
	$\int_{\frac{1}{2}}^{1}-\frac{3}{x}\mathrm{d}x=\left[-3\ln x\right]_{\frac{1}{2}}^{1}$	**M1**	For attempt to integrate, must have ln
	$=3\ln\frac{1}{2}$	**M1**	correct application of limits, dep on previous M mark
	$=-3\ln 2$	**M1**	realisation that value of integral is negative and making the adjustment
		M1	application of log law, dep on previous M mark
	Area $=9+\ln 8$	**A1**	

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