Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Integration

Time allowed: 1 hour 50 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 4, 8-10 (Paper 1): no calculator.
Questions 1-3, 5-7 (Paper 2): calculator allowed.

Total marks

/ 73

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 73 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 **(i)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{5}{3x+2}\right)$. [2]

(ii) Use your answer to part **(i)** to find $\int \frac{30}{(3x+2)^2}\,\mathrm{d}x$. [2]

(iii) Hence evaluate $\int_1^2 \frac{30}{(3x+2)^2}\,\mathrm{d}x$. [2]

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Differentiation, tangents, normals and stationary points Click for more variations

1(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{5}{3x+2}\right)=-5(3x+2)^{-2}\times 3$	**M1**	$-5(3x+2)^{-2}$
		A1	$\times 3$
1(ii)	$\int \frac{30}{(3x+2)^2}\,\mathrm{d}x=\left[\frac{-10}{(3x+2)}\right]$	**M1**	$\frac{1}{(3x+2)}$
		A1	$\times -10$
1(iii)	$\left[\frac{-10}{(3x+2)}\right]_1^2=-\frac{10}{8}+\frac{10}{5}$	**M1**	Insert limits and subtract
	$=\frac{3}{4}$	**A1**	

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2 Find the exact value of $\int_2^4 \frac{(x+1)^2}{x^2}\mathrm{d}x$. [6]

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Integration and definite integrals Click for more variations

2	$\frac{(x+1)^2}{x^2}=\frac{x^2+2x+1}{x^2}=1+\frac{2}{x}+\frac{1}{x^2}$	**2**	**B1** for expanding numerator seen anywhere. **M1** for attempt to divide *their* three term numerator by x^2.
	$\int 1+\frac{2}{x}+\frac{1}{x^2}\mathrm{d}x=x+2\ln x-\frac{1}{x}+(c)$	**2**	**A2/1/0** minus 1 each error or omission.
	$\left[4-2\ln 4-\frac{1}{4}\right]-\left[2+2\ln 2-\frac{1}{2}\right]$	**M1**	**Dep** insert 4 and 2 into *their* three or two term integrand and subtract correctly.
	$=\frac{9}{4}+2\ln 2$	**A1**	oe must be exact two terms. isw

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3 **(a)** Integrate the following with respect to x.

(i) e^{5x-2} [2]

(ii) $\frac{1}{4-3x}$ where $x < \frac{4}{3}$ [2]

(b) Show that $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sec^2\left(\frac{1}{2}x\right) dx = 2\left(1 - \frac{\sqrt{3}}{3}\right)$. [3]

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Integration and definite integrals Click for more variations

3(a)(i)	$\frac{1}{5}e^{5x-2}(+c)$	**2**	**B1** for ke^{5x-2} where k is a non-zero constant
3(a)(ii)	$-\frac{1}{3}\ln(4-3x)(+c)$	**2**	**B1** for $-\frac{1}{3}\ln 4-3x$ or $kln(4-3x)$ where k is a non-zero constant
3(b)	$2\tan\left(\frac{1}{2}x\right)$	**M2**	**M1** for $k\tan\left(\frac{1}{2}x\right)$ where k is a non-zero constant
	Fully justified completion to given answer e.g. $2\tan\frac{\pi}{4}-2\tan\frac{\pi}{6}=2\left(1-\frac{\sqrt{3}}{3}\right)$	**A1**	Or $2(\tan\frac{\pi}{4}-\tan\frac{\pi}{6})=2\left(1-\frac{\sqrt{3}}{3}\right)$

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4 It is given that $y = (3x+1)^2 \ln(3x+1)$.

(a) Find $\frac{dy}{dx}$. [3]

(b) Hence find $\int (3x+1)\ln(3x+1)\,dx$. [4]

Integration and definite integrals Click for more variations

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4(a)	$\frac{dy}{dx} = 2\times 3(3x+1)\ln(3x+1) + \frac{3(3x+1)^2}{3x+1}$ Simplified to: $\frac{dy}{dx} = 3(3x+1)(1+2\ln(3x+1))$	**3**	**B1** for $\frac{3}{3x+1}$ **M1** for attempt to differentiate a product **A1** for all terms other than $\frac{3}{3x+1}$ correct. Allow unsimplified.
4(b)	$k(3x+1)^2\ln(3x+1)$ soi	**B1**	
	$\int(3x+1)dx = \frac{3x^2}{2} + x\ (+c)$	**B1**	May be a multiple May be seen as $\frac{1}{3}\times\frac{1}{2}\times(3x+1)^2$
	$\frac{1}{6}(3x+1)^2\ln(3x+1) - \frac{3x^2}{4} - \frac{x}{2} + c$	**B2**	**B1** for two correct algebraic terms For **B2** must have (+ c)

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5 A curve is such that $\frac{dy}{dx} = (2x + 1)^{\frac{1}{2}}$. The curve passes through the point (4, 10).

(i) Find the equation of the curve. [4]

(ii) Find $\int y dx$ and hence evaluate $\int_0^{1.5} y dx$. [5]

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5 **(i)**	$[y=]\dfrac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}}(+c)$ oe	**B2**	or **B1** for $(2x+1)^{\frac{1}{2}+1}$
	$10=\dfrac{2}{6}(2(4)+1)^{\frac{3}{2}}+c$ oe	**M1**	for valid attempt to find c; condone slips e.g. omission of power or sign error
	$y=\dfrac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}}+c$ seen and $c = 1$ or	**A1**	must have y = ….; condone f(x) = ...
	$y=\dfrac{(2x+1)^{\frac{3}{2}}}{2\times\frac{3}{2}}+1$ isw		
(ii)	$\int\left(\dfrac{1}{3}(2x+1)^{\frac{3}{2}}+1\right)dx=\dfrac{1}{15}(2x+1)^{\frac{5}{2}}+x(+const)$	**B1 + B1**	**B1** for $(2x+1)^{\frac{3}{2}+1}$, **B1** for $\dfrac{1}{15}(2x+1)^{\frac{5}{2}}$
		B1ft	**B1 ft** *their c* from **(i)** provided $c \neq 0$
	$\left[\dfrac{1}{15}(2x+1)^{\frac{5}{2}}+x\right]_0^{1.5}=$		
	$\left[\dfrac{1}{15}(2(1.5)+1)^{\frac{5}{2}}+(1.5)\right]-\left[\dfrac{1}{15}(2(0)+1)^{\frac{5}{2}}+0\right]$	**M1**	for a genuine attempt to find F(1.5) – F(0) in an attempt to integrate *their y*; if their F(0) is 0 must see at least their F(1.5)– 0; condone + c as long as their c is **not** numerical.
	$\dfrac{107}{30}$ oe isw	**A1**	if decimal 3.57 or more accurate e.g. 3.566

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6 A curve is such that $\frac{d^2y}{dx^2} = \cos\left(4x - \frac{\pi}{4}\right)$. Given that $\frac{dy}{dx} = \frac{3}{4}$ at the point $\left(\frac{3\pi}{16}, \frac{\pi}{4}\right)$ on the curve, find the equation of the curve. [7]

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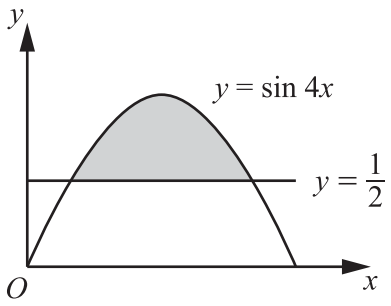
Integration and definite integrals Click for more variations

6	$\int \cos\left(4x-\frac{\pi}{4}\right)\mathrm{d}x = \frac{1}{4}\sin\left(4x-\frac{\pi}{4}\right)(+c)$	**B2**	**B1** for $k\sin\left(4x-\frac{\pi}{4}\right)$ where $k>0$ or $k=-\frac{1}{4}$
	$\frac{3}{4}=\frac{1}{4}\sin\left(4\left(\frac{3\pi}{16}\right)-\frac{\pi}{4}\right)+c$	**M1**	**FT** *their k* providing **B1** awarded
	$-\frac{1}{16}\cos\left(4x-\frac{\pi}{4}\right)+\frac{1}{2}x \quad (+A)$	**2**	**M1 FT** for $m\cos\left(4x-\frac{\pi}{4}\right)+\left(their\,\frac{1}{2}\right)x \quad (+A)$ **FT** $their\,k\sin\left(4x-\frac{\pi}{4}\right)+their\,c$ providing at least **B1 M1** awarded
	$y=-\frac{1}{16}\cos\left(4x-\frac{\pi}{4}\right)+\frac{1}{2}x+\frac{5\pi}{32}$ oe, cao	**2**	**M1 FT** for $\frac{\pi}{4}=-\frac{1}{16}\cos\left(\frac{3\pi}{4}-\frac{\pi}{4}\right)+\frac{1}{2}\left(\frac{3\pi}{16}\right)+A$ **FT** $(their\,m)\cos\left(4x-\frac{\pi}{4}\right)+\left(their\,\frac{1}{2}\right)x+A$ providing previous **M1** awarded

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7 **(a)** Solve the equation $\sin 4x = \frac{1}{2}$ for $0 \leqslant x \leqslant \frac{\pi}{4}$, giving your answers in terms of π. [2]

(b)



The diagram shows parts of the graphs of $y = \sin 4x$ and $y = \frac{1}{2}$.
Find the exact area of the shaded region enclosed by the curve and the line. [5]

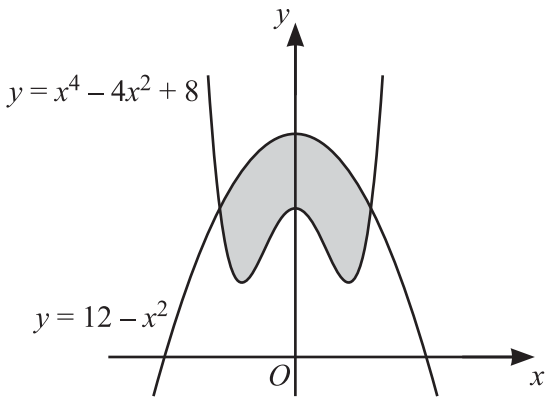
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Areas by integration Click for more variations

7(a)	$[x =]\ \frac{\pi}{24}, \frac{5\pi}{24}$ and no extras in range	**2**	**B1** for $[x =]\ \frac{\pi}{24}$ or $\frac{5\pi}{24}$ ignoring extras or for $4x = \frac{\pi}{6}$ **and** $\frac{5\pi}{6}$ soi
7(b)	Correct and complete plan soi e.g. $\int_{their\frac{\pi}{24}}^{their\frac{5\pi}{24}} \left(\sin(4x) - \frac{1}{2}\right) dx$ or $2\int_{their\frac{\pi}{24}}^{\frac{\pi}{8}} \left(\sin(4x) - \frac{1}{2}\right) dx$	**M1**	**FT** *their* limits in radians from part (a) or $\int_{their\frac{\pi}{24}}^{their\frac{5\pi}{24}} \sin 4x\,dx - \frac{1}{2}\left(their\frac{5\pi}{24} - their\frac{\pi}{24}\right)$
	Integrates sin 4x: $-\frac{1}{4}\cos 4x$	**B2**	**B1** for $k\cos 4x$ where $k < 0$ or $k = \frac{1}{4}$
	Substitutes exact limits in correct order: $-\frac{1}{4}\cos 4\left(\frac{5\pi}{24}\right) - \left[-\frac{1}{4}\cos 4\left(\frac{\pi}{24}\right)\right]$ oe soi	**M1**	**FT** *their* exact limits and *their* $-\frac{1}{4}\cos 4x$ providing at least **B1** awarded
	$\frac{\sqrt{3}}{4} - \frac{\pi}{12}$ or exact equivalent	**A1**	

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8



The diagram shows part of each of the curves $y = 12 - x^2$ and $y = x^4 - 4x^2 + 8$.

Find the area of the shaded region enclosed by the two curves. [7]

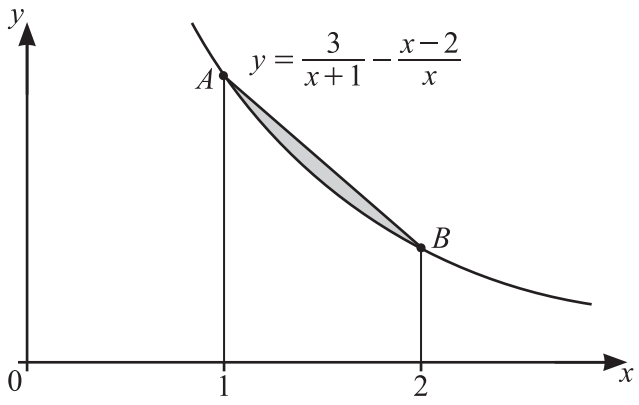
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8	$-x^4 + 3x^2 + 4\ (= 0)$	**M1**	For attempt to solve simultaneously Allow one term error
	$(x^2 - 4)(x^2 + 1) = 0$ oe	**M1**	**Dep** for an attempt to solve *their* 3-term quadratic equation in terms of x^2
	$x = 2$ and -2 only	**A1**	
	$\int(-x^4 + 3x^2 + 4)\mathrm{d}x$	**M1**	For an attempt to integrate *their* quadratic–quartic or the other way around with one term correctly integrated Allow without limits
	$\left[-\frac{x^5}{5} + x^3 + 4x\right]_{-2}^{2}$	**A1**	All correct with correct limits of 2 and -2 or with limits of 0 and 2 or -2 and 0 then double the area of the integral
	$-\frac{32}{5} + 8 + 8 - (\frac{32}{5} - 8 - 8)$	**M1**	**Dep** for correct use of *their* limits into *their* integral substituted correctly If *their* integration – must see substitution of limits to award the mark
	$= \frac{96}{5}$ or 19.2	**A1**	

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9



The diagram shows part of the curve $y = \frac{3}{x+1} - \frac{x-2}{x}$.

The points A and B lie on the curve such that the x-coordinate of A is 1 and the x-coordinate of B is 2.

(a) Find the y-coordinates of A and B. [1]

(b) Show that the area of the shaded region enclosed by the line AB and the curve is $\frac{a}{4} - \ln\frac{b}{2}$, where a and b are integers to be found. [7]

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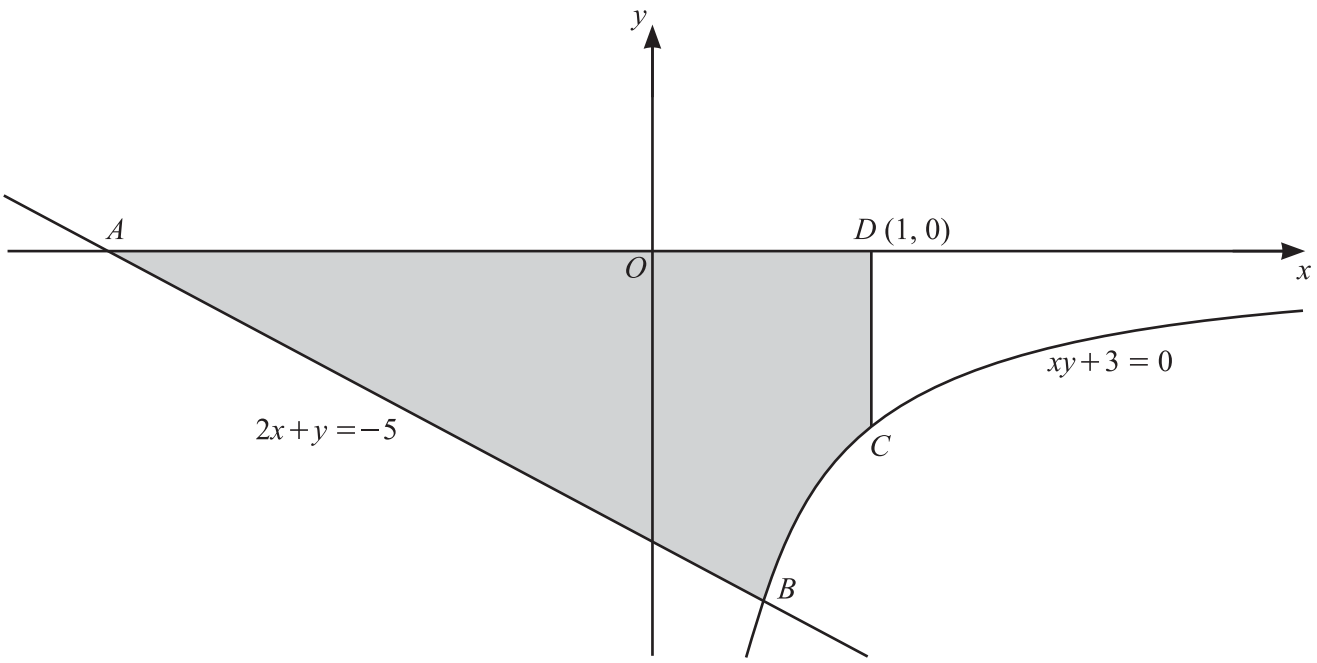
Areas by integration Click for more variations

9(a)	$\frac{5}{2}$, 1	**B1**	

9(b)	Area of trapezium = $\frac{7}{4}$	**B1**	**FT** on *their y* coordinates
	Area under curve = $\int_1^2 \left(\frac{3}{x+1} - 1 + \frac{2}{x}\right) \mathrm{d}x$	**B1**	Condone omission of limits here
	$3\ln(x+1) - x + 2\ln x$	**2**	**M1** for at least one correct log term
	$(3\ln 3 - 2 - 2\ln 2) - (3\ln 2 - 1)$	**M1**	**Dep** on previous **M** mark
	$\ln\frac{27}{2} - 1$	**M1**	**Dep** on previous **M** mark for simplification using log rules correctly
	$\frac{11}{4} - \ln\left(\frac{27}{2}\right)$	**A1**	

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10



The diagram shows the straight line $2x+y=-5$ and part of the curve $xy+3=0$. The straight line intersects the x-axis at the point A and intersects the curve at the point B. The point C lies on the curve. The point D has coordinates $(1, 0)$. The line CD is parallel to the y-axis.

(a) Find the coordinates of each of the points A and B. [3]

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(b) Find the area of the shaded region, giving your answer in the form $p + \ln q$, where p and q are positive integers. [6]

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Areas by integration Click for more variations

10(a)	$A\left(-\frac{5}{2},\ 0\right)$	**B1**	
	$x(-5-2x)+3=0$ $2x^2+5x-3=0$ $(2x-1)(x+3)=0$	**M1**	For attempt to eliminate one variable, obtain a 3-term quadratic equation = 0 and attempt to solve
	$B\left(\frac{1}{2},\ -6\right)$	**A1**	Allow A1 if just the x-coordinates or just the y-coordinates are given
10(b)	Area of triangle $=\frac{1}{2}\left(\frac{5}{2}+\frac{1}{2}\right)\times 6\ ,=9$	**M1**	For attempt at triangle using *their* values
	$\int_{\frac{1}{2}}^{1}-\frac{3}{x}\mathrm{d}x=\left[-3\ln x\right]_{\frac{1}{2}}^{1}$	**M1**	For attempt to integrate, must have ln
	$=3\ln\frac{1}{2}$	**M1**	correct application of limits, dep on previous M mark
	$=-3\ln 2$	**M1**	realisation that value of integral is negative and making the adjustment
		M1	application of log law, dep on previous M mark
	Area $=9+\ln 8$	**A1**	

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