Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Logarithmic and exponential functions

Time allowed: 1 hour 50 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 1, 3, 5, 9 (Paper 1): no calculator.
Questions 2, 4, 6-8, 10 (Paper 2): calculator allowed.

Total marks

/ 71

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 71 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 The population, P, of a certain bacterium t days after the start of an experiment is modelled by $P = 800e^{kt}$, where k is a constant.

(i) State what the figure 800 represents in this experiment. [1]

(ii) Given that the population is 20 000 two days after the start of the experiment, calculate the value of k. [3]

(iii) Calculate the population three days after the start of the experiment. [2]

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1(i)	The number of bacteria at the start of the experiment	**B1**	
1(ii)	$20\,000 = 800e^{kt}$ so $\frac{20\,000}{800} = e^{2k}$ or $\ln 20\,000 = \ln 800 + \ln(e^{2k})$	**M1**	use of given equation and attempt to solve for e^{2k} or use logs correctly
	$2k = \ln 25$	**M1**	correct method to obtain $2k$
	1.61	**A1**	
1(iii)	$P = 800e^{3\ln 5}$	**M1**	Substitution of $t = 3$ in formula using *their k*
	$= 100\,000$	**A1**	answer in range 99800 to 100200

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2 Solve the equation $64^{x+\frac{1}{3}} + 2^{3x} - 3 = 0$. [4]

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2	$4\times 8^{2x}+8^{x}-3[=0]$ or $4\times(2^{3x})^{2}+2^{3x}-3[=0]$ oe	**B1**	
	$\left(4(8^{x})-3\right)\left(8^{x}+1\right)$ or $\left(4(2^{3x})-3\right)\left(2^{3x}+1\right)$ oe	**M1**	**FT** *their* 3-term quadratic in 8^{x} or 2^{3x}
	$8^{x}=\frac{3}{4}$ or $2^{3x}=\frac{3}{4}$ oe, nfww	**A1**	
	$x=\log_{8}(0.75)$ or $\frac{1}{3}\log_{2}(0.75)$ or –0.138[34…] oe	**A1**	and no other solutions

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3 Solve the equation $\log_2(x+1) - 4\log_{(x+1)}2 = 3$. [5]

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3	Correct change of base e.g. $\log_2(x+1)-\dfrac{4\log_2 2}{\log_2(x+1)}=3$ or $\dfrac{\log_{(x+1)}(x+1)}{\log_{(x+1)}2}-4\log_{(x+1)}2=3$	**B1**	
	$\left(\log_2(x+1)\right)^2-3\log_2(x+1)-4 \quad [=0]$ oe or $4\left(\log_{(x+1)}2\right)^2+3\log_{(x+1)}2-1 \quad [=0]$ oe	**B1**	
	Factorises or solves e.g. $\left(\log_2(x+1)-4\right)\left(\log_2(x+1)+1\right)=0$ or $\left(4\log_{(x+1)}2-1\right)\left(\log_{(x+1)}2+1\right)=0$	**M1**	**FT** *their* 3-term quadratic in a suitable logarithm providing correct change of base seen
	$\log_2(x+1)=their 4$ and $\log_2(x+1)=their(-1)$ or $\log_{(x+1)}2=their\dfrac{1}{4}$ and $\log_{(x+1)}2=their(-1)$ **and** correctly solves as far as x = … at least once	**M1**	**dep** on previous **M1** **FT** $\log_2(x+1)=a$ or $\log_{(x+1)}2=b$ where a and b are constants
	$x=15,\ x=-\dfrac{1}{2}$ nfww mark final answer	**A1**	

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4 The profit $\$P$ made by a company in its nth year is modelled by

$$P = 1000\mathrm{e}^{an+b}.$$

In the first year the company made \$2000 profit.

(i) Show that $a + b = \ln 2$. [1]

In the second year the company made \$3297 profit.

(ii) Find another linear equation connecting a and b. [2]

(iii) Solve the two equations from parts **(i)** and **(ii)** to find the value of a and of b. [2]

(iv) Using your values for a and b, find the profit in the 10th year. [2]

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4 **(i)**	$2000 = 1000e^{a+b} \rightarrow a + b = \ln 2$	**B1**	
(ii)	$3297 = 1000e^{2a-b} \rightarrow 2a + b$ $= \ln 3.297$ oe	**M1** **A1**	substitution of 2, 3297 and rearrange
(iii)	Solve for one value $a = 0.5$ and $b = 0.193$ or 0.19	**M1** **A1**	
(iv)	$n = 10$ $P = 1000e^{5.193}$ $= \$180\,000.$	**M1** **A1**	

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5 **(a)** Given that $\log_2 x + 2\log_4 y = 8$, find the value of xy. [3]

(b) Using the substitution $y = 2^x$, or otherwise, solve $2^{2x+1} - 2^{x+1} - 2^x + 1 = 0$. [4]

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5(a)	Either $2\log_4 y = \log_2 y$ Or $\log_2 x = 2\log_4 x$	**B1**	
	Either $\log_2 x + \log_2 y = 8$ leading to $\log_2 xy = 8$ Or $2\log_4 x + 2\log_4 y = 8$ leading to $\log_4 xy = 4$	**M1**	For use of log law
	$xy = 256$	**A1**	
5(b)	$2y^2 - 3y + 1 = 0$	**B1**	
	$y = \frac{1}{2}, 1$	**M1**	For attempt to solve for y
	$x = -1$	**A1**	
	$x = 0$	**A1**	

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6 **(a)** Solve the equation $5^{2y-1} = 6 \times 3^{y}$, giving your answer correct to 3 decimal places. [3]

(b) Solve the equation $e^{2x} - 4 + 3e^{-2x} = 0$, giving your answers in exact form. [4]

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6(a)	$(2y-1)\log 5 = \log 6 + y\log 3$ oe or $2y\log 5 = \log 30 + y\log 3$ oe OR [rearranges $\frac{5^{2y}}{5} = 6\times 3^{y}$ and collects powers to a single power in y] $\left(\frac{5^2}{3}\right)^{y} = 30$ oe	**M1**	
	Collects terms and factorises: $y(2\log 5 - \log 3) = \log 6 + \log 5$ oe or $y(2\log 5 - \log 3) = \log 30$ oe OR takes logs $y = \log_{\frac{25}{3}} 30$ oe or $y\log\left(\frac{5^2}{3}\right) = \log 30$ oe	**M1**	**FT** if of equivalent difficulty
	1.604	**A1**	cao

6(b)	$e^{4x} - 4e^{2x} + 3 = 0$ or $(e^{2x})^2 - 4e^{2x} + 3 = 0$ oe	**M1**	condone one error
	Factorises: $(e^{2x}-1)(e^{2x}-3)$ oe or solves $e^{4x} - 4e^{2x} + 3 = 0$ oe	**M1**	**FT** $a(e^{2x})^2 + be^{2x} + c[=0]$
	$e^{2x} = 1, \quad e^{2x} = 3$	**A1**	
	$x = 0$, $x = \frac{1}{2}\ln 3$ or exact equivalent	**A1**	

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7 **(a)** Solve the equation $\log_6(2x-3)=\frac{1}{2}$. Give your answer in exact form. [2]

(b) Solve the equation $\ln 2u-\ln(u-4)=1$. Give your answer in exact form. [3]

(c) Solve the equation $\frac{3^v}{27^{2v-5}}=9$. [3]

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7(a)	$2x-3=6^{\frac{1}{2}}$ oe, soi	**M1**	
	$x=\dfrac{6^{\frac{1}{2}}+3}{2}$ or $x=\dfrac{\sqrt{6}+3}{2}$	**A1**	
7(b)	$\ln\dfrac{2u}{u-4}=\ln\mathrm{e}$ soi or $\ln\dfrac{2u}{u-4}=1$ soi or $\ln 2u=\ln\mathrm{e}(u-4)$ soi	**M1**	Condone one sign or bracketing error
	$\dfrac{2u}{u-4}=\mathrm{e}$ or $2u=\mathrm{e}(u-4)$ oe	**M1**	**FT** *their* logarithmic equation
	$u=\dfrac{4\mathrm{e}}{\mathrm{e}-2}$ or $u=\dfrac{-4\mathrm{e}}{2-\mathrm{e}}$ or equivalent exact form	**A1**	
7(c)	$\dfrac{3^v}{\left(3^3\right)^{2v-5}}=3^2$ oe soi or $\dfrac{9^{\frac{v}{2}}}{\left(9^{\frac{3}{2}}\right)^{2v-5}}=9$ oe soi or $\log 3^v-\log 27^{2v-5}=\log 9$ oe soi	**B1**	
	$15-5v=2$ oe or $v\log 3-(2v-5)\log 27=\log 9$	**M1**	**FT** *their* exponential equation in the same base or *their* logarithmic equation with any consistent base, providing *their* exponential or logarithmic equation has at most one sign or arithmetic error
	$v=\dfrac{13}{5}$ oe	**A1**	

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8 **(a)** Solve the equation $\dfrac{9^{5x}}{27^{x-2}} = 243$. [3]

(b) $\log_a \sqrt{b} - \dfrac{1}{2} = \log_b a$, where $a > 0$ and $b > 0$.

Solve this equation for b, giving your answers in terms of a. [5]

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8(a)	$\frac{3^{10x}}{3^{3x-6}}[=243]$ oe or $\log 9^{5x} - \log 27^{x-2} = \log 243$ oe	**B1**	
	$3^{7x+6} = 3^5$ soi oe or $5x(\log 9) - (x-2)\log 27 = \log 243$	**M1**	
	$x = -\frac{1}{7}$	**A1**	

8(b)	$\frac{1}{2}\log_a b - \frac{1}{2} = \frac{1}{\log_a b}$ or $\frac{\frac{1}{2}}{\log_b a} - \frac{1}{2} = \log_b a$	**B2**	**B1** for bringing down the power of $\frac{1}{2}$ e.g. $\frac{1}{2}\log_a b$ or for a change of base e.g. $\frac{1}{\log_a b}$
	Clears the fraction and rearranges $\frac{1}{2}(\log_a b)^2 - \frac{1}{2}\log_a b = 1$ oe $(\log_a b)^2 - \log_a b - 2 = 0$ oe or let $x = \log_a b$ $x^2 - x - 2 = 0$ oe or $\frac{1}{2} - \frac{1}{2}\log_b a = (\log_b a)^2$ $0 = 2(\log_b a)^2 + \log_b a - 1$ oe or let $y = \log_b a$ $2y^2 + y - 1 = 0$	**M1**	
	$(\log_a b - 2)(\log_a b + 1)$ oe or $(2\log_b a - 1)(\log_b a + 1)$	**M1**	
	[$\log_a b = 2$, $\log_a b = -1$ or $\log_b a = \frac{1}{2}$, $\log_b a = -1$ leading to] $b = a^2$, $b =$ oe	**A1**	

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9 **(a)** Express $2+3\lg x-\lg y$ as a single logarithm to base 10. [3]

(b) **(i)** Solve $6x+7-\frac{3}{x}=0$. [2]

(ii) Hence, given that $6\log_a 3+7-3\log_3 a=0$, find the possible values of a. [4]

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9(a)	$\lg 100 = 2$	**B1**	
	$3\lg x = \lg x^3$	**B1**	
	$\lg \frac{100x^3}{y}$	**B1**	
9(b)(i)	$6x^2 + 7x - 3 = 0$ $(2x+3)(3x-1) = 0$	**M1**	For obtaining in suitable quadratic form and attempt to solve
	$x = -\frac{3}{2}$ $x = \frac{1}{3}$	**A1**	For both

9(b)(ii)	$x = \log_a 3$ $\frac{1}{3} = \log_a 3$	**M1**	For realising connection with part **(i)** and attempt to solve $\frac{1}{3} = \log_a 3$ or $-\frac{3}{2} = \log_a 3$
	$a = 27$	**A1**	
	$-\frac{3}{2} = \log_a 3$	**M1**	For realising connection with part **(i)** and attempt to solve $-\frac{3}{2} = \log_a 3$ or $\frac{1}{3} = \log_a 3$
	$a = \left(\frac{1}{3}\right)^{\frac{2}{3}}$ or 0.481or $\left(\frac{1}{9}\right)^{\frac{1}{3}}$ oe	**A1**	

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10 **(a)** **(i)** Solve $\frac{5^{2x+3}}{25^{2x}} = \frac{25^{2-x}}{125^{x}}$. [3]

(ii) Solve $1g\ y + 1g(y - 15) = 2$. [4]

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(b) Without using a calculator, and showing each stage of your working, find the value of

$2\log_{12} 4 - \frac{1}{2}\log_{12} 81 + 4\log_{12} 3$. [3]

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10	**(a)(i)** Express in powers of 5. $\left(\frac{5^{2x+3}}{5^{4x}} = \frac{5^{2(2-x)}}{5^{3x}}\right)$	M1
	Use rules of indices $(2x + 3 - 4x = 2(2 - x) - 3x)$	M1
	$\frac{1}{3}$	A1
	(ii) LHS = $\lg y(y - 15)$	B1
	$2 = \lg 100$	B1
	Solve 3 term quadratic	M1
	20 only	A1
	(b) $\log_{12} 16 - \log_{12} 9 + \log_{12} 81$	B1
	Correctly combine 3 logarithms	M1
	2	A1
		[10]

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