**Write your name, centre number and candidate number in the boxes below.**

| Surname | Other names |
|---|---|
| | |

Centre number | | | | | |

Candidate number | | | | |

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# Permutations and combinations

Time allowed: 1 hour 45 minutes

**Subject code** **0606**

# Additional Mathematics

## Questions from Paper 1 and Paper 2

**You must have:** a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
**Questions 1, 6-10 (Paper 1): no calculator.**
**Questions 2-5 (Paper 2): calculator allowed.**

Total marks

**/ 67**

### Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

### Information

- This paper has 10 questions worth 67 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

### Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

**1** Solve the equation $(n-4)\,{}^{n+1}C_5 = {}^{n+2}C_7$. [3]

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| | | | |
|---|---|---|---|
| **1** | $\dfrac{(n-4)(n+1)!}{(n-4)!5!} = \dfrac{(n+2)!}{(n-5)!7!}$<br>leading to<br>$7\times 6 = n+2$ | **2** | **B1** for $7\times 6$ or 42 seen as a result of simplification of the numerical factorials<br><br>**B1** for $n+2$ seen as a result of simplification of the algebraic factorials |
| | $n = 40$ | **B1** | |

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**2** **(a)** In an examination, candidates must select 2 questions from the 5 questions in section A and select 4 questions from the 8 questions in section B. Find the number of ways in which this can be done. [2]

**(b)** The digits of the number 6 378 129 are to be arranged so that the resulting 7-digit number is even. Find the number of ways in which this can be done. [2]

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| | | | |
|---|---|---|---|
| **2**(a) | ${}^5C_2 \times {}^8C_4$ oe | **M1** | |
| | 700 | **A1** | |
| **2**(b) | $3 \times 6!$ oe | **M1** | |
| | 2160 | **A1** | |

**3** In a singing competition there are 8 contestants. Each contestant sings in the first round of this competition.

**(i)** In how many different orders could the contestants sing? [1]

After the first round 5 contestants are chosen.

**(ii)** In how many different ways can these 5 contestants be chosen? [2]

These 5 contestants sing again and then First, Second and Third prizes are awarded to three of them.

**(iii)** In how many different ways can the prizes be awarded? [2]

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**3** **(i)** 40320 B1

**(ii)** $\frac{8\times7\times6\times5\times4}{5\times4\times3\times2(\times1)}$ or $\frac{8!}{5!\times3!}$ M1

56 A1

**(iii)** uses 5, 4 and 3 only M1

60 A1 [5]

**4** A sports club has the following members.

5 runners, 4 swimmers, 3 gymnasts

**(a)** These members stand in a straight line.

Find the number of ways that this can be done when all the runners stand together, all the swimmers stand together and all the gymnasts stand together. [2]

**(b)** Four of these members are selected for an event.

Find the number of ways that this can be done when at least one runner, at least one swimmer and at least one gymnast must be selected. [3]

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| | | | |
|---|---|---|---|
| **4**(a) | 103 680 | **2** | **M1** for $k \times 5! \times 4! \times 3!$ where $k = 1, 2, 3, 6$ |
| **4**(b) | 270 | **3** | **M2** for<br>${}^5C_1 \times {}^4C_2 \times {}^3C_1 + {}^5C_1 \times {}^4C_1 \times {}^3C_2 +$<br>${}^5C_2 \times {}^4C_1 \times {}^3C_1$<br>oe<br><br>OR<br><br>**M1** for at least one correct product<br><br>If 0 scored then **SC1** for a <u>final</u> answer of 540<br>or 264 |

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**5** **(a)** There are 3 girls and 2 boys standing in a straight line. Find the number of possible orders in each of the following cases.

**(i)** No girls are next to each other. [2]

**(ii)** The 2 boys are not next to each other. [2]

**(b)** 12 people, including Anjie and Bubay, are divided into 3 groups of 4 people. Anjie and Bubay must not be in the same group.

Find the number of ways in which the 3 groups can be selected. [2]

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| | | | |
|---|---|---|---|
| **5**(a)(i) | 12 | **2** | **B1** for $3! \times 2!$ or ${}^2P_2 \times {}^3P_3$ oe |
| **5**(a)(ii) | 72 | **2** | **B1** for $5! - 4! \times 2!$ oe<br>or $6 \times 2! \times 3!$ oe |
| **5**(b) | 4200 | **2** | **B1** for ${}^{10}C_3 \times {}^7C_3 \left[ \times {}^4C_4 \right]$ oe<br>or ${}^{10}C_4 \times {}^6C_3 \left[ \times {}^3C_3 \right]$ oe |

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**6** In this question $n \geqslant 6$ .
Use an algebraic method to show that ${}^{n}C_{5} - {}^{n-1}C_{5}$ can be written as ${}^{n-1}C_{4}$. [4]

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| | | | |
|---|---|---|---|
| **6** | $\frac{n!}{(n-5)!5!}-\frac{(n-1)!}{(n-6)!5!}$<br>$\frac{(n-1)!\times[n-(n-5)]}{(n-5)!5!}$ | **M1** | For substituting ${}^{n}C_5$ and ${}^{n-1}C_5$ |
| | $\frac{(n-1)!}{(n-6)!5!}\left(\frac{n}{n-5}-1\right)$ oe | **M1** | **dep** on previous **M1** awarded<br>For Factorising $\frac{(n-1)!}{(n-6)!5!}$ or $\frac{(n-1)!}{(n-5)!5!}$<br>Do not allow if using ${}^{n-1}C_4$ to simplify the left-hand side |
| | $\frac{(n-1)!}{(n-6)!5!}\left(\frac{5}{n-5}\right)$ oe | **M1** | **dep** on at least one previous **M1** awarded<br>Simplifying fraction |
| | Completes argument:<br>$\frac{(n-1)!}{(n-6)!5!}\left(\frac{5}{n-5}\right)=\frac{(n-1)!}{(n-1-4)!4!}={}^{n-1}C_4$ | **A1** | |
| | **Alternative 1** | | |
| | $\frac{n!}{(n-5)!5!}-\frac{(n-1)!}{(n-6)!5!}$ | **(M1)** | For substituting ${}^{n}C_5$ and ${}^{n-1}C_5$ |
| | $\frac{1}{(n-6)!}=\frac{n-5}{(n-5)!}$ or $n!=n\times(n-1)!$ | **(B1)** | **dep** on previous **M1** awarded |
| | $\frac{(n-1)!\times[n-(n-5)]}{(n-5)!5!}$ | **(M1)** | **dep** on previous **M1** awarded<br>factorising $(n-1)!$ in the fraction |
| | $\frac{(n-1)!\times5}{(n-5)!5!}=\frac{(n-1)!}{(n-5)!4!}={}^{n-1}C_4$ | **(A1)** | |
| | **Alternative 2** | | |
| | $\frac{n!}{(n-5)!5!}-\frac{(n-1)!}{(n-6)!5!}$ | **(M1)** | For substituting ${}^{n}C_5$ and ${}^{n-1}C_5$ could be implied by expansions of the factorial |
| | $\frac{n(n-1)(n-2)(n-3)(n-4)}{5!}-\frac{(n-1)(n-2)(n-3)(n-4)(n-5)}{5!}$ | **(B1)** | **dep** on previous **M1** awarded |
| | $\frac{(n-1)(n-2)(n-3)(n-4)[n-(n-5)]}{5!}$ | **(M1)** | **dep** on previous **M1** awarded<br>factorising $(n-1)(n-2)(n-3)(n-4)$ in the fraction |
| **6** | Completes argument:<br>$\frac{(n-1)(n-2)(n-3)(n-4)}{4!}=\frac{(n-1)!}{4!(n-5)!}={}^{n-1}C_4$ | **(A1)** | |

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**7** **(a)** A 4-digit code is to be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. No digit may be used more than once in any code. A code may start with 0.

**(i)** Find how many codes can be formed. [1]

**(ii)** Find how many codes form an odd number. [1]

**(iii)** Find how many codes form a number greater than 1000. [2]

**(b)** A team of 9 people is to be chosen from a group of 15 people. The group includes a family of 4 people who must not be separated. Find the number of teams that can be chosen. [3]

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| | | | |
|---|---|---|---|
| **7**(a)(i) | 5040 | **B1** | |
| **7**(a)(ii) | 2520 | **B1** | |
| **7**(a)(iii) | There are 504 codes less than 1000<br>$5040-504=4536$ | **2** | **M1** for *their* **(i)** $-504$ or $9\times$(a product of 3 relevant numbers) |
| **7**(b) | With family: 462 | **B1** | |
| | Without family: 55 | **B1** | |
| | Total: 517 | **B1** | |

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**8** **(a)** A 5-character password is to be formed from the following 10 characters.

| | |
|---|---|
| Letters | A B C X Y Z |
| Symbols | * $ # & |

No character can be used more than once in any 5-character password.

**(i)** Find the number of passwords that can be formed. [1]

**(ii)** Find the number of passwords that can be formed if the password has to contain at least one symbol. [2]

**(iii)** Find the number of passwords that can be formed if the password has to start with two letters and end with two symbols. [2]

**(b)** A team of 8 people is to be chosen from 5 doctors, 4 teachers and 6 police officers.

Find how many possible teams have the same number of doctors as teachers. [5]

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| | | | |
|---|---|---|---|
| **8**(a)(i) | 30 240 | **B1** | |
| **8**(a)(ii) | 720 | **B1** | the number of passwords with no symbols.<br>Not part of a product |
| | 29 520 | **B1** | |
| **8**(a)(iii) | $(6\times5)\times6\times(4\times3)=2160$ oe | **2** | **B1** for $(6\times5)$ and $(4\times3)$ soi |

| | | | |
|---|---|---|---|
| **8**(b) | 1 of each and 6 police officers = 20 | **B1** | For ${}^4C_1\times{}^5C_1\times{}^6C_6$ must be evaluated, could be implied by a correct total |
| | 2 of each and 4 police officers = 900 | **B1** | For ${}^4C_2\times{}^5C_2\times{}^6C_4$ must be evaluated, could be implied by a correct total |
| | 3 of each and 2 police officers = 600 | **B1** | For ${}^4C_3\times{}^5C_3\times{}^6C_2$ must be evaluated, could be implied by a correct total |
| | 4 of each and no police officers = 5 | **B1** | For ${}^4C_4\times{}^5C_4$ must be evaluated, could be implied by a correct total |
| | Total = 1525 | **B1** | |

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**9** **(a)** A 5-digit number is to be formed from the seven digits 1, 2, 3, 5, 6, 8 and 9. Each digit can only be used once in any 5-digit number. Find the number of different 5-digit numbers that can be formed if

**(i)** there are no restrictions, [1]

**(ii)** the number is divisible by 5, [1]

**(iii)** the number is greater than 60 000, [1]

**(iv)** the number is greater than 60 000 and even. [3]

**(b)** Ranjit has 25 friends of whom 15 are boys and 10 are girls. Ranjit wishes to hold a birthday party but can only invite 7 friends. Find the number of different ways these 7 friends can be selected if

**(i)** there are no restrictions, [1]

**(ii)** only 2 of the 7 friends are boys, [1]

**(iii)** the 25 friends include a boy and his sister who cannot be separated. [3]

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| | | | |
|---|---|---|---|
| **9**(a)(i) | 2520 | **B1** | |
| **9**(a)(ii) | 360 | **B1** | |
| **9**(a)(iii) | 1080 | **B1** | |
| **9**(a)(iv) | 6 or 8 to start with<br>No of ways = $2\times5\times4\times3\times2$<br>= 240 | **B1** | |
| | 9 to start with<br>No of ways = $1\times5\times4\times3\times3$<br>= 180 | **B1** | |
| | Total number of ways = 420 | **DB1** | Dependent on both previous B marks |

| | | | |
|---|---|---|---|
| **9**(a)(iv) | **Alternative 1**<br><br>All numbers > 6000 – all odd numbers > 6000 | **B1** | plan and attempt to use, must be using 1080 |
| | $1080-180-480$ | **B1** | for 180 and 480 |
| | Total number of ways = 420 | **DB1** | Dependent on both previous B marks |
| | **Alternative 2**<br><br>Even numbers > 60000 : Odd numbers > 60000<br>7 : 11 | **B1** | correct ratio |
| | Total number of ways = $\frac{7}{18}\times1080$ | **B1** | |
| | = 420 | **DB1** | Dependent on both previous B marks |
| **9**(b)(i) | 480700 | **B1** | |
| **9**(b)(ii) | 26460 | **B1** | |
| **9**(b)(iii) | With brother and sister $^{23}C_5 = 33649$ | **B1** | for $^{23}C_5$ or $^{23}C_5\times{}^{k}C_k$ |
| | Without brother and sister $^{23}C_7 = 245157$ | **B1** | for $^{23}C_7$ or $^{23}C_7\times{}^{k}C_k$ |
| | Total number of ways = 278806 | **B1** | for $^{23}C_5 + {}^{23}C_7$ and evaluation |

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**10** **(a)** 10 people are to be chosen, to receive concert tickets, from a group of 8 men and 6 women.

**(i)** Find the number of different ways the 10 people can be chosen if 6 of them are men and 4 of them are women. [2]

The group of 8 men and 6 women contains a man and his wife.

**(ii)** Find the number of different ways the 10 people can be chosen if both the man and his wife are chosen or neither of them is chosen. [3]

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**(b)** Freddie has forgotten the 6-digit code that he uses to lock his briefcase. He knows that he did not repeat any digit and that he did not start his code with a zero.

**(i)** Find the number of different 6-digit numbers he could have chosen. [1]

Freddie also remembers that his 6-digit code is divisible by 5.

**(ii)** Find the number of different 6-digit numbers he could have chosen. [3]

Freddie decides to choose a new 6-digit code for his briefcase once he has opened it. He plans to have the 6-digit number divisible by 2 and greater than 600 000, again with no repetitions of digits.

**(iii)** Find the number of different 6-digit numbers he can choose. [3]

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| | | | |
|---|---|---|---|
| **10**(a)(i) | ${}^{8}C_{6} \times {}^{6}C_{4}$ | **B1** | either ${}^{8}C_{6}$ or ${}^{6}C_{4}$ |
| | 420 | **B1** | |
| **10**(a)(ii) | ${}^{12}C_{8} + {}^{12}C_{10}$ | **B2** | B1 for each |
| | = 561 | **B1** | |
| | Alternate scheme:<br>$1001 - \left(2 \times {}^{12}C_{9}\right)$ | **B1 B1** | |
| | = 561 | **B1** | |
| **10**(b)(i) | 136 080 | **B1** | |
| **10**(b)(ii) | No of ways ending with 0 - 15 120 | **B1** | |
| | No of ways ending with 5 - 13 440 | **B1** | |
| | Total 28 560 | **B1** | |
| **10**(b)(iii) | Starting with 6 or 8 - 13 440 | **B1** | |
| | Starting with 7 or 9 - 16 800 | **B1** | |
| | Total = 30 240 | **B1** | |

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