Mark scheme

Radians: arc length and sector area

Additional Mathematics 0606 · Questions from Paper 1 and Paper 2

Total marks

73

MARKS FOR EACH QUESTION

1	2	3	4	5	6	7	8	9	10	Total
4	6	7	7	7	7	8	8	8	11	**73**

Question 1 **[4]**

1	**(i)**	$500 = \frac{1}{2}r^2(1.6)$	**M1**	
		25 only	**A1**	± 25 is **A0**
	(ii)	*their* 25 + *their* 25 + *their* 25 × 1.6 or better	**M1**	*their* 25 must be positive
		90	**A1**	

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Question 2 **[6]**

2(i)	$9.6 = 12\theta$	**M1**	For use of arc length
	$\theta = 0.8$	**A1**	

2(ii)	Either $\tan\theta = \frac{AB}{12}$, $(AB = 12.36)$ Or $OB = \frac{12}{\cos\theta}$ $(OB = 17.22)$	**M1**	For attempt to find AB or OB using *their* θ May be implied by a correct triangle area Allow if using degrees consistently
	Either Area $\Delta OAB = \frac{1}{2} \times 12 \times$ *their* 12.36 Or Area $\Delta OAB = \frac{1}{2} \times 12 \times$ *their* $17.22 \times \sin\theta$ $(= 74.1$ or $74.2)$	**M1**	Allow if using degrees consistently For attempt to find area of triangle using *their* θ
	Area of sector $OAC = \frac{1}{2} \times 12^2 \times 0.8$ $= 57.6$	**B1**	Allow unsimplified
	Area of shaded region = 16.5 or 16.6	**A1**	

Question 3 **[7]**

3(i)	$\frac{1}{2}r^2\theta = 48$	**B1**	
	$\theta = \frac{96}{r^2}$	**B1**	**Dep** Must have previous B1
3(ii)	$r\theta = 12$	**B1**	For statement of arc length
	$r \times \frac{96}{r^2} = 12$	**M1**	For attempt to use part **(i)** to find either r or θ
	$r = 8$ and $\theta = 1.5$	**A1**	For both

3(iii)	Area $= 48 - \left(\frac{1}{2}r^2\sin\theta\right)$	**M1**	For a complete method including a correct attempt for the area of the triangle using their r and θ
	=16.1	**A1**	

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Question 4 **[7]**

4(a)	$27 = 12\theta$ $\theta = \frac{9}{4}$ oe	**B1**	$\angle AOB = \theta$
	Either $\tan(\pi - \theta) = \frac{CB}{12}$ soi Or $\frac{CB}{\sin(\pi - \theta)} = \frac{12}{\sin\left(\theta - \frac{\pi}{2}\right)}$	**M1**	Allow with *their* θ.
	Perimeter = $24 + 27 + 2(14.86..)$	**M1**	Allow with *their CB*.
	Perimeter = awrt 80.7	**A1**	From correct working only
4(b)	$\left(\frac{1}{2} \times 12^2 \times \textit{their } \theta\right) + (12 \times \textit{their } CB)$ 340 oe or 341 oe	**3**	**M1** for each area **A1** for awrt 340 or 341

Question 5 **[7]**

5(a)	$ACB = 2\tan^{-1}\left(\frac{12}{5}\right)$ oe	**M1**	
	$ACB = (2 \times 1.176\ldots)$ = 2.35 to 2 dp	**A1**	Must see justification to 2 dp
5(b)	Arc length = $5 \times ACB$	**B1**	
	Perimeter = 35.8	**B1**	Allow awrt 35.8
5(c)	Area = $(12 \times 5) - \left(\frac{1}{2} \times 5^2 \times 2.35\right)$	**M2**	**M1** for area of kite or area of sector **M1 dep** for kite area – sector area
	30.6	**A1**	Allow greater accuracy Any use of fractions gets **A0**

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Question 6 **[7]**

6(a)	$6.5\left(\frac{3\pi}{8}\right)+5.2\left(\frac{3\pi}{8}\right)+2(6.5-5.2)$	**M2**	**M1** for $6.5\left(\frac{3\pi}{8}\right)$ or $5.2\left(\frac{3\pi}{8}\right)$
	16.38 to 16.4	**A1**	
6(b)	[Angle PRQ =] 2ϕ soi	**B1**	
	$y = 2a\cos\phi$ oe or $y=\frac{a\sin(\pi-2\phi)}{\sin\phi}$ oe $y^2 = a^2+a^2-2a^2\cos(\pi-2\phi)$ oe or $y^2 = a^2+a^2+2a^2\cos(2\phi)$ oe	**B1**	
	Complete and correct plan soi: $\pi a^2-\frac{1}{2}(2a\cos\phi)^2(2\phi)$ oe or $\pi a^2-\frac{1}{2}\left(\frac{a\sin(\pi-2\phi)}{\sin\phi}\right)^2(2\phi)$ oe or $\pi a^2-\frac{1}{2}(a^2+a^2-2a^2\cos(\pi-2\phi))(2\phi)$ oe or $\pi a^2-\frac{1}{2}(a^2+a^2+2a^2\cos(2\phi))(2\phi)$	**M1**	**FT** *their* 2ϕ and *their* expression for y or y^2 in terms of a and ϕ
	$a^2\left(\pi-4\phi\cos^2\phi\right)$ or $\pi a^2-\frac{a^2\phi\sin^2(\pi-2\phi)}{\sin^2\phi}$ or $\pi a^2-2\phi(a^2-a^2\cos(\pi-2\phi))$ or $\pi a^2-2\phi(a^2+a^2\cos 2\phi)$ oe	**A1**	

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Question 7 **[8]**

7(a)	$\frac{1}{2}\times 10^2\times\theta = 35$ so $\theta = 0.7$	**B1**	

7(b)	Arc length CD: 7	**B1**	
	$\sin(0.35) = \frac{AB/2}{12}$	**M1**	For a complete method to find AB, could be using cosine rule
	$AB = 8.23(0)$	**A1**	
	Perimeter = $7 + 4 + 8.23 = 19.2$	**A1**	
7(c)	Area of triangle $= \frac{1}{2}12^2 \sin 0.7$	**M1**	For complete attempt at triangle area, may use equivalent method
	Area of triangle $= 46.4$	**A1**	
	Shaded area $= 11.4$	**A1**	Follow through on *their* area of the triangle

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Question 8 **[8]**

8(a)	$\frac{1}{2}\times 24^2\times\theta = 432$	**M1**	
	$\theta = \frac{3}{2}$ rads soi	**A1**	
	$24 \times$ *their* θ	**M1**	
	36 cao	**A1**	
	Alternative method		
	$s = r\theta$ soi and $\frac{1}{2}\times r\times s = 432$	**(B1)**	
	$\frac{1}{2}\times 24\times s = 432$	**(M1)**	
	$s = \frac{432\times 2}{24}$ oe	**(M1)**	
	$[s =]$ 36	**(A1)**	
8(b)(i)	$[OB =]$ $2y\cos\alpha$ oe	**B1**	
8(b)(ii)	$\frac{(their\, 2y\cos\alpha)\times y\sin\alpha}{2}$ $-\frac{1}{2}\times(their\, y\cos\alpha)^2\times\alpha$ oe	**M2**	**M1** for either area
	correct completion to $\frac{y^2}{2}\cos\alpha(2\sin\alpha - \alpha\cos\alpha)$	**A1**	

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Question 9 **[8]**

9(a)	$\frac{\pi}{3}$	**2**	**M1** for angle AOB: $\frac{2\pi}{3}$ or angle AQO: $\frac{\pi}{6}$ or angle OAB: $\frac{\pi}{6}$ or e.g. cos $(AOB) = -\frac{1}{2}$ or $\sin\left(\frac{1}{2}AOB\right) = \frac{\sqrt{3}}{2}$ or AQ or $BQ = 2\sqrt{3}$ If 0 scored, **SC1** for answer of 1.047 rads
9(b)	$2\sqrt{3} - \frac{2\pi}{3}$ or $\frac{6\sqrt{3} - 2\pi}{3}$ oe nfww	**6**	**B5** correct calculation that simplifies to give shaded area e.g. $\left(\frac{4\pi}{3} - \sqrt{3}\right) - \left(2\pi - 3\sqrt{3}\right)$ OR **B1** for sector AOB: $\frac{1}{2} \times 2^2 \times \frac{2\pi}{3}$ oe **B1** for ΔAOB or ΔAOQ or ΔBOQ: $\frac{1}{2} \times 2^2 \times \sin\frac{2\pi}{3}$ or $\frac{1}{2} \times 2 \times 2\sqrt{3} \sin\frac{\pi}{6}$ oe **B1** for sector AQB: $\frac{1}{2} \times \left(2\sqrt{3}\right)^2 \times \frac{\pi}{3}$ oe **B1** for ΔAQB: $\frac{1}{2}\left(2\sqrt{3}\right)^2 \sin\frac{\pi}{3}$ oe OR **B2 FT** *their* exact angles for a correct calculation for any two of these areas OR **B2** for sector AOB: 4.188; ΔAOB or ΔAOQ or ΔBOQ: 1.732; sector AQB: 6.282; ΔAQB: 5.195[5] from using 1.047 rads OR **B1FT** *their* exact angles for a correct calculation for any one of these areas OR **B1** for one of these areas using 1.047 rads

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Question 10 **[11]**

10(i)	$\frac{1}{2}\times 12^2\times\theta = 150$	**M1**	use of sector area
	$\theta = 2.083$, so $\theta = 2.08$ to 2dp	**A1**	
10(ii)	Area of triangle $AOB = \frac{1}{2}\times 12^2 \sin 2.08$	**M1**	correct method for area of triangle
	Area of segment $= 150 - \frac{1}{2}\times 12^2 \times \sin 2.08$	**A1**	allow unsimplified, using $\theta = 2.08,\ 2.083$ or $\frac{150}{72}$
	$\sin 1.04 = \frac{\frac{AB}{2}}{12}$	**M1**	correct trigonometric statement using $\theta = 2.08,\ 2.083$ or $\frac{150}{72}$ with attempt to obtain AB
	AB = awrt 20.7	**A1**	
	Shaded area = *their* $AB\times 8$ − *their* segment area	**M1**	execution of a correct 'plan'(rectangle – segment)
	awrt 78.4 or 78.5	**A1**	
10(iii)	Arc AB = 25 or 24.96	**B1**	
	Perimeter= 25 + *their* AB + 16	**M1**	correct 'plan' (arc + *their* AB + 2×8)
	awrt 61.7	**A1**	

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