Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Radians: arc length and sector area

Time allowed: 1 hour 50 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 2-5, 7, 10 (Paper 1): no calculator.
Questions 1, 6, 8-9 (Paper 2): calculator allowed.

Total marks

/ 73

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 73 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

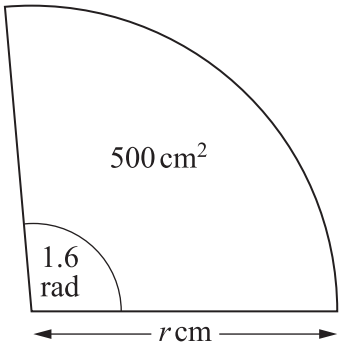
Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1



The diagram shows a sector of a circle of radius r cm. The angle of the sector is 1.6 radians and the area of the sector is $500\,\text{cm}^2$.

(i) Find the value of r. [2]

(ii) Hence find the perimeter of the sector. [2]

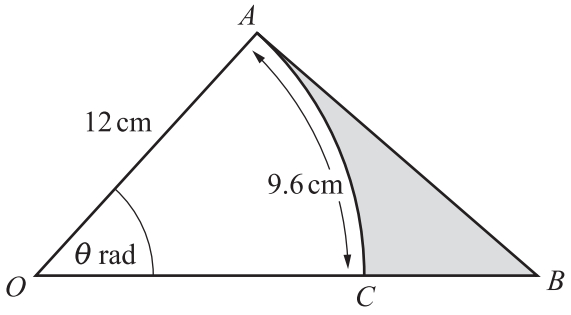
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Circular measure Click for more variations

1	**(i)**	$500 = \frac{1}{2}r^2(1.6)$	**M1**	
		25 only	**A1**	± 25 is **A0**
	(ii)	*their* 25 + *their* 25 + *their* 25 × 1.6 or better	**M1**	*their* 25 must be positive
		90	**A1**	

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2



The diagram shows the right-angled triangle OAB. The point C lies on the line OB. Angle $OAB = \frac{\pi}{2}$ radians and angle $AOB = \theta$ radians. AC is an arc of the circle, centre O, radius 12 cm and AC has length 9.6 cm.

(i) Find the value of θ. [2]

(ii) Find the area of the shaded region. [4]

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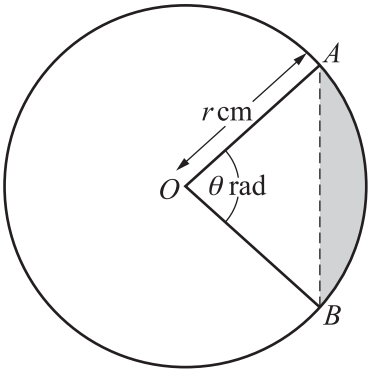
Circular measure Click for more variations

2(i)	$9.6 = 12\theta$	**M1**	For use of arc length
	$\theta = 0.8$	**A1**	

2(ii)	Either $\tan\theta = \frac{AB}{12}$, $\left(AB = 12.36\right)$ Or $OB = \frac{12}{\cos\theta}$ $\left(OB = 17.22\right)$	**M1**	For attempt to find AB or OB using *their* θ May be implied by a correct triangle area Allow if using degrees consistently
	Either Area $\Delta OAB = \frac{1}{2} \times 12 \times$ *their* 12.36 Or Area $\Delta OAB = \frac{1}{2} \times 12 \times$ *their* $17.22 \times \sin\theta$ $\left(= 74.1 \text{ or } 74.2\right)$	**M1**	Allow if using degrees consistently For attempt to find area of triangle using *their* θ
	Area of sector $OAC = \frac{1}{2} \times 12^2 \times 0.8$ $= 57.6$	**B1**	Allow unsimplified
	Area of shaded region = 16.5 or 16.6	**A1**	

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3



The diagram shows a circle with centre O and radius r cm. The minor arc AB is such that angle AOB is θ radians. The area of the minor sector AOB is 48 cm^2.

(i) Show that $\theta = \frac{96}{r^2}$. [2]

(ii) Given that the minor arc AB has length 12 cm, find the value of r and of θ. [3]

(iii) Using your values of r and θ, find the area of the shaded region. [2]

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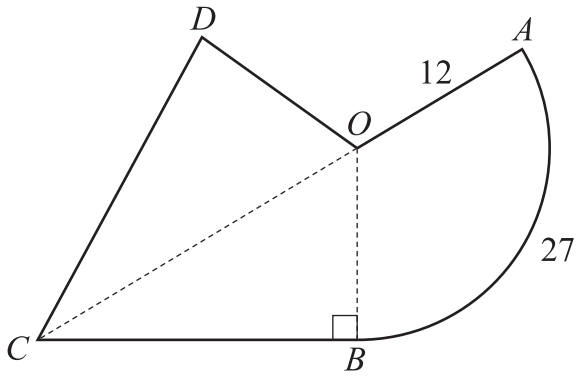
Circular measure Click for more variations

3(i)	$\frac{1}{2}r^2\theta = 48$	**B1**	
	$\theta = \frac{96}{r^2}$	**B1**	**Dep** Must have previous B1
3(ii)	$r\theta = 12$	**B1**	For statement of arc length
	$r \times \frac{96}{r^2} = 12$	**M1**	For attempt to use part **(i)** to find either r or θ
	$r = 8$ and $\theta = 1.5$	**A1**	For both

3(iii)	Area = $48 - \left(\frac{1}{2}r^2 \sin\theta\right)$	**M1**	For a complete method including a correct attempt for the area of the triangle using their r and θ
	=16.1	**A1**	

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4 In this question all lengths are in centimetres and all angles are in radians.



The diagram shows a badge which consists of a minor sector, OAB, of the circle with centre O and radius 12, and a kite $OBCD$, where $OB = OD$ and $CD = CB$. The arc AB has length 27. The line OB is perpendicular to the line CB, and COA is a straight line.

(a) Find the perimeter of the badge. [4]

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(b) Find the area of the badge. [3]

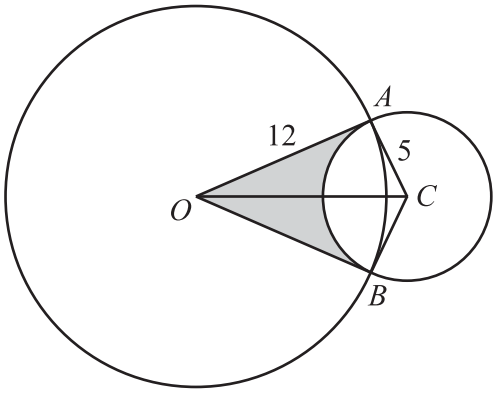
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Circular measure Click for more variations

4(a)	$27 = 12\theta$ $\theta = \frac{9}{4}$ oe	**B1**	$\angle AOB = \theta$
	Either $\tan(\pi - \theta) = \frac{CB}{12}$ soi Or $\frac{CB}{\sin(\pi - \theta)} = \frac{12}{\sin\left(\theta - \frac{\pi}{2}\right)}$	**M1**	Allow with *their* θ.
	Perimeter = $24 + 27 + 2(14.86..)$	**M1**	Allow with *their CB*.
	Perimeter = awrt 80.7	**A1**	From correct working only
4(b)	$\left(\frac{1}{2} \times 12^2 \times \textit{their } \theta\right) + (12 \times \textit{their } CB)$ 340 oe or 341 oe	**3**	**M1** for each area **A1** for awrt 340 or 341

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5 In this question, all lengths are in centimetres and all angles are in radians.



The diagram shows a circle with centre O and radius 12, and a circle with centre C and radius 5. The circles intersect at the points A and B, such that OA and OB are tangents to the circle with centre C.

(a) Show that the obtuse angle ACB is 2.35 radians, correct to 2 decimal places. [2]

(b) Find the perimeter of the shaded region. [2]

(c) Find the area of the shaded region. [3]

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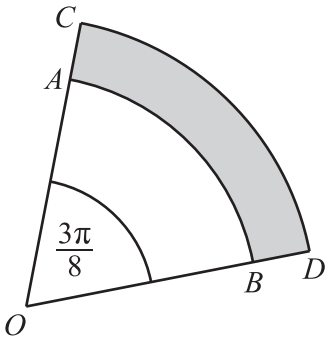
Circular measure Click for more variations

5(a)	$ACB = 2\tan^{-1}\left(\frac{12}{5}\right)$ oe	**M1**	
	$ACB = (2\times 1.176\ldots)$ = 2.35 to 2 dp	**A1**	Must see justification to 2 dp
5(b)	Arc length = $5\times ACB$	**B1**	
	Perimeter = 35.8	**B1**	Allow awrt 35.8
5(c)	Area = $(12\times 5)-\left(\frac{1}{2}\times 5^2\times 2.35\right)$	**M2**	**M1** for area of kite or area of sector **M1 dep** for kite area – sector area
	30.6	**A1**	Allow greater accuracy Any use of fractions gets **A0**

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6 In this question, all lengths are in centimetres and all angles are in radians.

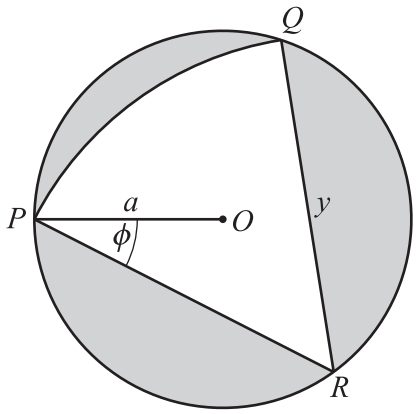
(a)



The diagram shows sectors AOB and COD of two circles with the same centre, O. Angle AOB is $\frac{3\pi}{8}$ and the length of OC is 6.5. It is given that OAC and OBD are straight lines and $OA : OC$ is 4 : 5. Find the perimeter of the shaded region. [3]

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(b)



The diagram shows a circle with centre O and radius a. Sector PQR is a sector of a different circle with centre R and radius y. Angle OPR is ϕ. Find, in terms of a and ϕ only, the total area of the three shaded regions. Simplify your answer. [4]

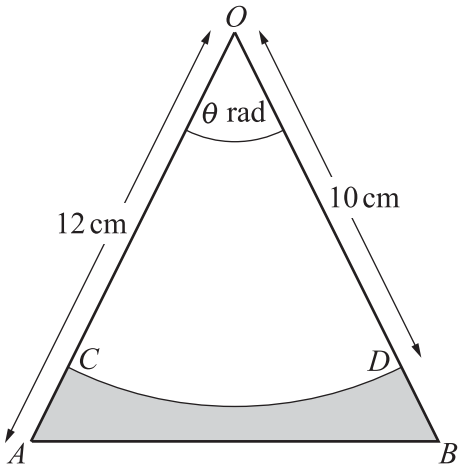
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Circular measure Click for more variations

6(a)	$6.5\left(\frac{3\pi}{8}\right)+5.2\left(\frac{3\pi}{8}\right)+2(6.5-5.2)$	**M2**	**M1** for $6.5\left(\frac{3\pi}{8}\right)$ or $5.2\left(\frac{3\pi}{8}\right)$
	16.38 to 16.4	**A1**	
6(b)	[Angle PRQ =] 2ϕ soi	**B1**	
	$y = 2a\cos\phi$ oe or $y=\frac{a\sin(\pi-2\phi)}{\sin\phi}$ oe $y^2 = a^2+a^2-2a^2\cos(\pi-2\phi)$ oe or $y^2 = a^2+a^2+2a^2\cos(2\phi)$ oe	**B1**	
	Complete and correct plan soi: $\pi a^2-\frac{1}{2}(2a\cos\phi)^2(2\phi)$ oe or $\pi a^2-\frac{1}{2}\left(\frac{a\sin(\pi-2\phi)}{\sin\phi}\right)^2(2\phi)$ oe or $\pi a^2-\frac{1}{2}(a^2+a^2-2a^2\cos(\pi-2\phi))(2\phi)$ oe or $\pi a^2-\frac{1}{2}(a^2+a^2+2a^2\cos(2\phi))(2\phi)$	**M1**	**FT** *their* 2ϕ and *their* expression for y or y^2 in terms of a and ϕ
	$a^2\left(\pi-4\phi\cos^2\phi\right)$ or $\pi a^2-\frac{a^2\phi\sin^2(\pi-2\phi)}{\sin^2\phi}$ or $\pi a^2-2\phi(a^2-a^2\cos(\pi-2\phi))$ or $\pi a^2-2\phi(a^2+a^2\cos 2\phi)$ oe	**A1**	

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7



The diagram shows an isosceles triangle OAB such that $OA = OB = 12\,\text{cm}$ and angle $AOB = \theta$ radians. Points C and D lie on OA and OB respectively such that CD is an arc of the circle, centre O, radius 10 cm. The area of the sector $OCD = 35\,\text{cm}^2$.

(a) Show that $\theta = 0.7$. [1]

(b) Find the perimeter of the shaded region. [4]

(c) Find the area of the shaded region. [3]

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Circular measure Click for more variations

7(a)	$\frac{1}{2}\times 10^2\times\theta = 35$ so $\theta = 0.7$	**B1**	

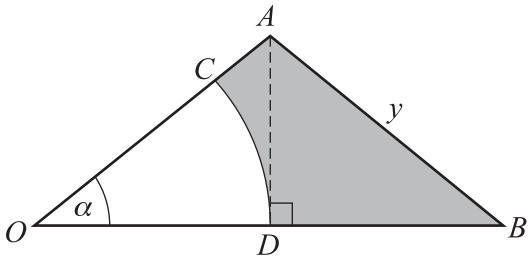
7(b)	Arc length CD: 7	**B1**	
	$\sin(0.35) = \frac{AB/2}{12}$	**M1**	For a complete method to find AB, could be using cosine rule
	$AB = 8.23(0)$	**A1**	
	Perimeter $= 7 + 4 + 8.23 = 19.2$	**A1**	
7(c)	Area of triangle $= \frac{1}{2}12^2 \sin 0.7$	**M1**	For complete attempt at triangle area, may use equivalent method
	Area of triangle $= 46.4$	**A1**	
	Shaded area $= 11.4$	**A1**	Follow through on *their* area of the triangle

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8 In this question all lengths are in centimetres and all angles are in radians.

(a) The area of a sector of a circle of radius 24 is $432\,\text{cm}^2$. Find the length of the arc of the sector. [4]

(b)



The diagram shows an isosceles triangle, OAB, with $AO = AB = y$ and height AD. OCD is a sector of the circle with centre O. Angle AOB is α.

(i) Find an expression for OB in terms of y and α. [1]

(ii) Hence show that the area of the shaded region can be written as $\frac{y^2}{2}\cos\alpha(2\sin\alpha - \alpha\cos\alpha)$. [3]

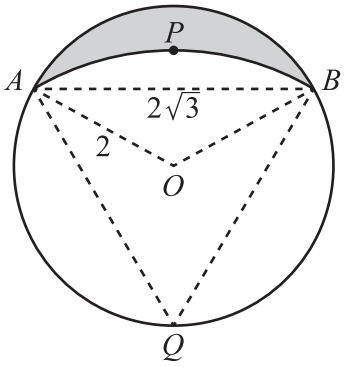
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Circular measure Click for more variations

8(a)	$\frac{1}{2}\times 24^2\times\theta = 432$	**M1**	
	$\theta = \frac{3}{2}$ rads soi	**A1**	
	$24 \times$ *their* θ	**M1**	
	36 cao	**A1**	
	Alternative method		
	$s = r\theta$ soi and $\frac{1}{2}\times r\times s = 432$	**(B1)**	
	$\frac{1}{2}\times 24\times s = 432$	**(M1)**	
	$s = \frac{432\times 2}{24}$ oe	**(M1)**	
	$[s =]$ 36	**(A1)**	
8(b)(i)	$[OB =]$ $2y\cos\alpha$ oe	**B1**	
8(b)(ii)	$\frac{(their\, 2y\cos\alpha)\times y\sin\alpha}{2}$ $-\frac{1}{2}\times(their\, y\cos\alpha)^2\times\alpha$ oe	**M2**	**M1** for either area
	correct completion to $\frac{y^2}{2}\cos\alpha(2\sin\alpha - \alpha\cos\alpha)$	**A1**	

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9 In this question the units are metres.



The diagram shows a circle, centre O and radius 2.
The chord AB has length $2\sqrt{3}$.
The point Q lies on the circle such that $AQ = BQ$.
The arc APB is part of a circle, centre Q.

(a) Find the exact value of angle AQB in radians. [2]

(b) Hence find the area of the shaded region. Give your answer in terms of π. [6]

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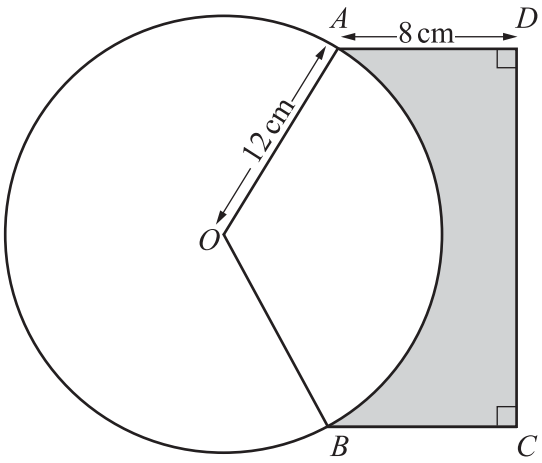
Circular measure Click for more variations

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9(a)	$\frac{\pi}{3}$	**2**	**M1** for angle AOB: $\frac{2\pi}{3}$ or angle AQO: $\frac{\pi}{6}$ or angle OAB: $\frac{\pi}{6}$ or e.g. cos $(AOB) = -\frac{1}{2}$ or $\sin\left(\frac{1}{2}AOB\right) = \frac{\sqrt{3}}{2}$ or AQ or $BQ = 2\sqrt{3}$ If 0 scored, **SC1** for answer of 1.047 rads
9(b)	$2\sqrt{3} - \frac{2\pi}{3}$ or $\frac{6\sqrt{3} - 2\pi}{3}$ oe nfww	**6**	**B5** correct calculation that simplifies to give shaded area e.g. $\left(\frac{4\pi}{3} - \sqrt{3}\right) - \left(2\pi - 3\sqrt{3}\right)$ OR **B1** for sector AOB: $\frac{1}{2} \times 2^2 \times \frac{2\pi}{3}$ oe **B1** for ΔAOB or ΔAOQ or ΔBOQ: $\frac{1}{2} \times 2^2 \times \sin\frac{2\pi}{3}$ or $\frac{1}{2} \times 2 \times 2\sqrt{3} \sin\frac{\pi}{6}$ oe **B1** for sector AQB: $\frac{1}{2} \times \left(2\sqrt{3}\right)^2 \times \frac{\pi}{3}$ oe **B1** for ΔAQB: $\frac{1}{2}\left(2\sqrt{3}\right)^2 \sin\frac{\pi}{3}$ oe OR **B2 FT** *their* exact angles for a correct calculation for any two of these areas OR **B2** for sector AOB: 4.188; ΔAOB or ΔAOQ or ΔBOQ: 1.732; sector AQB: 6.282; ΔAQB: 5.195[5] from using 1.047 rads OR **B1FT** *their* exact angles for a correct calculation for any one of these areas OR **B1** for one of these areas using 1.047 rads

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10



The diagram shows a circle, centre O, radius 12 cm. The points A and B lie on the circumference of the circle and form a rectangle with the points C and D. The length of AD is 8 cm and the area of the minor sector AOB is 150 cm^2.

(i) Show that angle AOB is 2.08 radians, correct to 2 decimal places. [2]

(ii) Find the area of the shaded region $ADCB$. [6]

(iii) Find the perimeter of the shaded region $ADCB$. [3]

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Circular measure Click for more variations

10(i)	$\frac{1}{2}\times 12^2\times\theta = 150$	**M1**	use of sector area
	$\theta = 2.083$, so $\theta = 2.08$ to 2dp	**A1**	
10(ii)	Area of triangle $AOB = \frac{1}{2}\times 12^2 \sin 2.08$	**M1**	correct method for area of triangle
	Area of segment = $150 - \frac{1}{2}\times 12^2 \times \sin 2.08$	**A1**	allow unsimplified, using $\theta = 2.08,\ 2.083$ or $\frac{150}{72}$
	$\sin 1.04 = \frac{\frac{AB}{2}}{12}$	**M1**	correct trigonometric statement using $\theta = 2.08,\ 2.083$ or $\frac{150}{72}$ with attempt to obtain AB
	AB = awrt 20.7	**A1**	
	Shaded area = *their* $AB\times 8$ – *their* segment area	**M1**	execution of a correct 'plan'(rectangle – segment)
	awrt 78.4 or 78.5	**A1**	

10(iii)	Arc AB = 25 or 24.96	**B1**	
	Perimeter= 25 + *their* AB + 16	**M1**	correct 'plan' (arc + *their* AB + 2×8)
	awrt 61.7	**A1**	

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