Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Remainder and factor theorem

Time allowed: 1 hour 20 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 1, 4, 6-7, 9 (Paper 1): no calculator.
Questions 2-3, 5, 8 (Paper 2): calculator allowed.

Total marks

/ 52

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 9 questions worth 52 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 The remainder when the expression $x^3 + kx^2 - 5x - 3$ is divided by $x - 2$ is 5 times the remainder when the expression is divided by $x + 1$. Find the value of k. [4]

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1	f(2) = 8 + 4k – 10 – 3 f(–1) = –1 + k + 5 – 3 (4k – 5) = 5(k + 1) leading to k = –10	M1 M1 M1 A1 [4]	M1 for use of x = 2 M1 for use of x = –1 M1 for attempt to link the two remainders

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2 It is given that $\mathrm{p}(x) = ax^3 - 7x^2 - bx + 9$, where a and b are constants.
$x - 3$ is a factor of $\mathrm{p}(x)$.
When $\mathrm{p}(x)$ is divided by $x + 2$ the remainder is -35.

Find the values of a and b. [5]

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Reduced to 98% to fit the page (wider than A4).

2	$27a - 63 - 3b + 9 = 0$ oe	**B1**	
	$-8a - 28 + 2b + 9 = -35$ oe	**B1**	
	Eliminates one unknown and solves for a or b	**M1**	**FT** *their* equations provided both equations are linear with terms in a and b
	$a = 2$, $b = 0$ nfww	**A2**	**A1** for each

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3 The expression $x^3 + 8x^2 + px - 25$ leaves a remainder of R when divided by $x - 1$ and a remainder of $-R$ when divided by $x + 2$.

(i) Find the value of p. [4]

(ii) Hence find the remainder when the expression is divided by $x + 3$. [2]

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3 (i)	f(1) = 1 + 8 + p – 25 (= p – 16) f(–2) = –8 + 32 – 2p – 25 (= –2p – 1) p – 16 = 2p + 1 oe p = –17	B1 B1 M1 A1	
(ii)	Evaluates f(–3) or divides by (x + 3) to remainder 71 (= 20 – 3p)	M1 A1√	**[6]**

4 The function f is such that f(x) = $4x^3 - 8x^2 + ax + b$, where a and b are constants. It is given that $2x - 1$ is a factor of f(x) and that when f(x) is divided by $x + 2$ the remainder is 20. Find the remainder when f(x) is divided by $x - 1$. [6]

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4	$f(-2): -2a + b = 84$	M1 A1	M1 for substitution of a correct value of x
	$f\left(\frac{1}{2}\right): \frac{1}{2}a + b = \frac{3}{2}$	A1	A1 for each correct equation (allow unsimplified)
	$a = -33,\ b = 18$	M1, A1	M1 for solution to obtain a and b
	$f(1) = -19$	√B1 [6]	√B1 on their a and b

5 It is given that $x - 1$ is a factor of f(x), where $f(x) = x^3 - 6x^2 + ax + b$.

(i) Express b in terms of a. [2]

(ii) Show that the remainder when f(x) is divided by $x - 3$ is twice the remainder when f(x) is divided by $x - 2$. [4]

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5 **(i)** f(1) = 0 M1
$b = 5 - a$ only A1

(ii) Finds f(2) and f(3) M1
substitute $b = 5 - a$ or $a = 5 - b$ M1
$2a - 22$ or $a - 11$ ($-2b - 12$ or $-b - 6$) A1
both correct A1 [6]

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6 The function $f(x) = ax^3 + 4x^2 + bx - 2$, where a and b are constants, is such that $2x - 1$ is a factor. Given that the remainder when $f(x)$ is divided by $x - 2$ is twice the remainder when $f(x)$ is divided by $x + 1$, find the value of a and of b. [6]

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6	$f\left(\frac{1}{2}\right) = \frac{a}{8} + 1 + \frac{b}{2} - 2$	M1	M1 for substitution of $x = \frac{1}{2}$ into f (x)
	leading to $a + 4b - 8 = 0$	A1	A1 for correct equation in any form
	f(2) = 2f(–1)	M1	M1 for attempt to substitute $x = 2$ **or** $x = -1$ into f(x) **and** use f(2) = ±2f(–1) or 2f(2) = ±f(–1)
	$8a + 16 + 2b - 2 = 2(-a + 4 - b - 2)$	A1	A1 for a correct equation in any form
	leading to $10a + 4b + 10 = 0$ or equivalent $\therefore a = -2,\ b = \frac{5}{2}$	DM1 A1 [6]	DM1 (on both previous M marks) for attempt to solve simultaneous equations to obtain either a or b A1 for both correct

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7 $p(x) = 2x^3 + 5x^2 + 4x + a$

$q(x) = 4x^2 + 3ax + b$

Given that $p(x)$ has a remainder of 2 when divided by $2x + 1$ and that $q(x)$ is divisible by $x + 2$,

(i) find the value of each of the constants a and b. [3]

Given that $r(x) = p(x) - q(x)$ and using your values of a and b,

(ii) find the exact remainder when $r(x)$ is divided by $3x - 2$. [3]

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7(i)	$\left(\mathrm{p}\left(-\frac{1}{2}\right)=\right)\ -\frac{1}{4}+\frac{5}{4}-2+a=2$ $\left(\mathrm{q}(-2)=\right)\ 16-6a+b=0$	**M1**	For either $\mathrm{p}\left(-\frac{1}{2}\right)=2$ or $\mathrm{q}(-2)=0$
	$a=3$	**A1**	
	$b=2$	**A1**	
7(ii)	$\mathrm{r}(x)=2x^3+x^2-5x+1$	**M1**	For $\mathrm{r}(x)$ using *their* $\mathrm{p}(x)$ and $\mathrm{q}(x)$
	$\mathrm{r}\left(\frac{2}{3}\right)=\frac{16}{27}+\frac{4}{9}-\frac{10}{3}+1$	**M1**	For $\mathrm{r}\left(\frac{2}{3}\right)$
	$=-\frac{35}{27}$	**A1**	Must be exact

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8 The polynomial $\mathrm{p}(x) = mx^3 - 29x^2 + 39x + n$, where m and n are constants, has a factor $3x - 1$, and remainder 6 when divided by $x - 1$. Show that $x - 2$ is a factor of $\mathrm{p}(x)$. [6]

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8	$\frac{m}{27}-\frac{29}{9}+\frac{39}{3}+n=0$ oe	**B1**	
	$m-29+39+n=6$ oe	**B1**	
	Eliminates one unknown correctly for a pair of linear equations in m and n and solves for one unknown	**M1**	
	$m=6$, $n=-10$	**A2**	**A1** for either
	[p(2) =] $48-116+78-10=0$ oe, nfww	**A1**	

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9 A polynomial $\mathrm{p}(x)$ is $ax^3 + 8x^2 + bx + 5$, where a and b are integers. It is given that $2x - 1$ is a factor of $\mathrm{p}(x)$ and that a remainder of –25 is obtained when $\mathrm{p}(x)$ is divided by $x + 2$.

(i) Find the value of a and of b. [5]

(ii) Using your values of a and b, find the exact solutions of $\mathrm{p}(x) = 5$. [2]

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9(i)	$\mathrm{p}\left(\frac{1}{2}\right)$: $\frac{a}{8}+2+\frac{b}{2}+5=0$	**M1**	substitution of $x=\frac{1}{2}$ and equating to zero (allow unsimplified)
	$\mathrm{p}(-2)$: $-8a+32-2b+5=-25$	**M1**	substitution of $x=-2$ and equating to –25 (allow unsimplified)
	leading to $a+4b+56=0$ $4a+b-31=0$ oe	**M1**	**M1dep** for solution of simultaneous equations to obtain a and b
	$a=12,\ b=-17$	**A2**	**A1** for each

9(ii)	$12x^3+8x^2-17x=0$ $x=0$	**B1**	for $x=0$
	$x=-\frac{1}{3}\pm\frac{\sqrt{55}}{6}$ oe	**B1**	

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