**Write your name, centre number and candidate number in the boxes below.**

| Surname | Other names |
|---|---|
| | |

Centre number | | | | | |

Candidate number | | | | |

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## Series

Time allowed: 2 hours 10 minutes

**Subject code** **0606**

# Additional Mathematics

**Questions from Paper 1 and Paper 2**

**You must have:** a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
**Questions 1-4 (Paper 1): no calculator.**
**Questions 5-10 (Paper 2): calculator allowed.**

Total marks

**/ 86**

### Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

### Information

- This paper has 10 questions worth 86 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

### Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

**1** **(i)** Find the first 3 terms of the expansion, in ascending powers of $x$, of $(1 + 3x)^6$. [2]

**(ii)** Hence find the coefficient of $x^2$ in the expansion of $(1 + 3x)^6 (1 – 3x – 5x^2)$. [3]

Binomial expansion Click for more variations

| | | | |
|---|---|---|---|
| **1** | **(i)** $1 + 18x + 135x^2 \ldots$ | B1,B1<br>[2] | B2, 1, 0 –1 for each error |
| | **(ii)** $(1 \times –5) + (18 \times –3) + (135 \times 1)$<br>$= 76$ | M1,A1ft<br>A1<br>[3] | M1 for a correct method using their **(i)**<br>A1ft on their 3 terms unsimplified |

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**2** **(i)** Find the first 3 terms, in descending powers of $x$, in the expansion of $\left(x + \frac{2}{x^2}\right)^6$. [3]

**(ii)** Hence find the term independent of $x$ in the expansion of $\left(2 - \frac{4}{x^3}\right)\left(x + \frac{2}{x^2}\right)^6$. [2]

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| | | |
|---|---|---|
| **2** **(i)** $\left(x+\frac{2}{x^2}\right)^6 = x^6 + 12x^3 + 60...$ | B3 | B1 for each correct term |
| | [3] | |
| **(ii)** Independent term = | M1 | M1 for sum of 2 products |
| $(2\times'60')+(-4\times'12') = 72$ | A1 | (2 × their 60)+(–4 × their 12) |
| | [2] | A1 for 72 |

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**3** The 7th and 10th terms of an arithmetic progression are 158 and 149 respectively.

**(a)** Find the common difference and the first term of the progression. [3]

**(b)** Find the least number of terms of the progression for their sum to be negative. [3]

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Arithmetic and geometric progressions Click for more variations

| | | | |
|---|---|---|---|
| **3**(a) | $a+6d=158$<br>$a+9d=149$ | **B1** | For both equations, may be implied by a correct $a$ and $d$ |
| | $d=-3,$ | **B1** | |
| | $a=176$ | **B1** | |

| | | | |
|---|---|---|---|
| **3**(b) | $\frac{n}{2}\left(352+\left(n-1\right)\left(-3\right)\right)\qquad\left(<0\right)$ | **M1** | For correct attempt at sum formula with *their a* and *their d*, |
| | $\frac{355}{3}$ or 118.3 oe | **A1** | |
| | 119 | **A1** | |

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**4** The coefficient of $x^2$ in the expansion of $(2 + px)^6$ is 60.

**(i)** Find the value of the positive constant $p$. [3]

**(ii)** Using your value of $p$, find the coefficient of $x^2$ in the expansion of $(3 - x)(2 + px)^6$. [3]

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| | | | |
|---|---|---|---|
| **4** | **(i)** $^6C_2\,(2^4)\,(px)^2$ or $\binom{6}{2}2^4(px)^2$ | B1 | Seen or implied, unsimplified |
| | $240p^2 = 60$ | M1 | M1 for their coefficient of $x^2 = 60$ and attempt to solve |
| | $p = \frac{1}{2}$ | A1 [3] | |
| | **(ii)** coefficients of the terms needed | M1 | M1 for realising that 2 terms are involved |
| | $(-1)\,{}^6C_1\,(2)^5\,p + (3 \times 60)$ | B1 | B1 for $(-1)\,{}^6C_1\,(2)^5\,p$ or $-192p$, using their $p$. |
| | $= 84$ | A1 [3] | |

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**5** The first three terms in the expansion of $(a+bx)^5(1+x)$ are $32-208x+cx^2$. Find the value of each of the integers $a$, $b$ and $c$. [7]

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| | | | |
|---|---|---|---|
| **5** | $a^5 + 5a^4bx + 10a^3b^2x^2$ | **2** | **B1** for powers or for coefficients |
| | $a^5 + \left(a^5 + 5a^4b\right)x + \left(10a^3b^2 + 5a^4b\right)x^2$ | **2** | **M1** for multiplying to obtain 5 terms<br>**A1** for all correct |
| | $a^5 = 32 \rightarrow a = 2$ | **A1** | |
| | $32 + 80b = -208 \rightarrow b = -3$ | **A1** | |
| | $10 \times 8 \times 9 + 5 \times 16 \times -3 = c \rightarrow c = 480$ | **A1** | |

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**6** The first three terms, in descending powers of $x$, in the expansion of $\left(3x^2 - a\right)^n \left(1 + \frac{1}{x^2}\right)^2$ can be written as $729x^{12} + 972x^{10} + bx^8$, where $a$, $b$ and $n$ are constants.

Find the values of $a$, $b$ and $n$. [9]

Binomial expansion Click for more variations

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| | | | |
|---|---|---|---|
| **6** | $n = 6$ stated; nfww | **B2** | **B1** for first term: $\left(3x^2\right)^n$ or $\left(3x^2\right)^6$ soi |
| | $-6\left(3x^2\right)^5 a + 2\left(729x^{10}\right) = 972x^{10}$ oe<br><br>and<br><br>$\frac{6\times5\left(3x^2\right)^4(-a)^2}{2} + 6\left(3x^2\right)^5 \times \frac{2}{x^2}(-a) + 729x^8 = bx^8$<br>oe | **B3** | **B2** for one correct equation<br><br>**OR**<br><br>**B1** for second term: $n(-a)\left(3x^2\right)^{n-1}$ or $6(-a)\left(3x^2\right)^5$ or $-1458ax^{10}$ soi<br><br>**B1** for third term: $\frac{n(n-1)(-a)^2\left(3x^2\right)^{n-2}}{2}$ or $\frac{6(5)(-a)^2\left(3x^2\right)^4}{2}$ or $1215a^2x^8$ soi<br><br>**B1** for $1 \ [+] \ \frac{2}{x^2} \ [+] \ \frac{1}{x^4}$ soi |
| | $a = \frac{1}{3}$ | **B2** | **B1** for $-1458a + 1458 = 972$ soi |
| | $b = -108$ | **B2** | **B1** for $1215a^2 - 2916a + 729$ soi |

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**7** **(a)** An arithmetic progression has twelve terms. The sum of the first three terms is $-36$ and the sum of the last three terms is 72. Find the first term and the common difference. [5]

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**(b)** The first three terms of a geometric progression are 1, 1.2 and 1.44. Find the smallest value of $n$ such that the sum of the first $n$ terms is greater than 500. [5]

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| | | | |
|---|---|---|---|
| **7**(a) | Correct pair of simplified linear equations in $a$ and $d$ with terms collected, e.g.,<br><br>$3a+3d=-36$ isw or $a+d=-12$ isw<br>$3a+30d=72$ isw or $a+10d=24$ isw | **B3** | **B2** for one correct simplified equation<br>or<br>**B1** for<br>$a+a+d+a+2d=-36$<br>or $\frac{3}{2}\{2a+(3-1)d\}=-36$<br>or $a+9d+a+10d+a+11d=72$<br>or $\frac{12}{2}\{2a+(12-1)d\}-\frac{9}{2}\{2a+(9-1)d\}=72$<br>or $12a+66d-9a-36d=72$<br>or $\frac{3}{2}\{2(a+9d)+(3-1)d\}=72$ |
| | Solves two linear equations for $d$ or $a$ e.g.<br>$27d=108 \rightarrow d=\ldots$<br>or $9d=36 \rightarrow d=\ldots$<br>or $a+10(-12-a)=24 \rightarrow a=\ldots$<br>$27a=-432 \rightarrow a=\ldots$ | **M1** | **FT** *their* linear equations in $a$ and $d$ providing at least **B1** earned and the equations have a solution |
| | $d=4$ and $a=-16$ nfww | **A1** | |
| **7**(b) | $1.2^n$ *101 | **B3** | where * is any inequality sign or =;<br><br>**B2** for $\frac{[1](1.2^n-1)}{(1.2-1)}*500$<br>or **B1** for $r=1.2$ soi |
| | $n\log 1.2*\log 101$ or $\log_{1.2}101$ soi | **M1** | **FT** $1.2^n$ * *their* 101 providing **B2** has been awarded and (*their* 101) $>0$ |
| | $n=26$ | **A1** | **dep** on all previous marks awarded |

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**8** **(a)** A geometric progression has first term 64 and common ratio 0.5.

**(i)** Find the 10th term. [2]

**(ii)** Find the sum of the first 10 terms. [2]

**(iii)** Find the sum to infinity. [1]

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**(b)** An arithmetic progression is such that $S_{20} - 400 = 2S_{10}$ and $u_1 : u_6$ is 1 : 5. Find the sum of the first 3 terms of this progression. [6]

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| | | | |
|---|---|---|---|
| **8**a)(i) | $\frac{1}{8}$ or 0.125 | **2** | **M1** for $64(0.5)^9$ oe |
| **8**(a)(ii) | $\frac{1023}{8}$ or 127.875 | **2** | **M1** for $\frac{64(1-0.5^{10})}{1-0.5}$ oe |
| **8**(a)(iii) | 128 | **1** | |
| **8**(b) | $\frac{20}{2}\{2a+19d\}-400=2\times\frac{10}{2}\{2a+9d\}$ oe, soi | **M2** | **M1** for $\frac{20}{2}\{2a+19d\}$ or $\frac{10}{2}\{2a+9d\}$ soi |
| | $5a = a + 5d$ soi | **M1** | |
| | $d = 4$ | **A1** | |
| | $a = 5$ | **A1** | |
| | 27 nfww | **B1** | must have earned all previous marks |

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**9** An arithmetic progression has first term $a$ and common difference $d$. The third term is 13 and the tenth term is 41.

**(a)** Find the value of $a$ and of $d$. [4]

**(b)** Find the number of terms required to give a sum of 2555. [4]

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**(c)** Given that $S_n$ is the sum to $n$ terms, show that $S_{2k} - S_k = 3k(1+2k)$. [4]

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| | | | |
|---|---|---|---|
| **9**(a) | Attempts to solve $a + 2d = 13$ and $a + 9d = 41$ oe | **M2** | **M1** for $a + 2d = 13$ and $a + 9d = 41$ soi |
| | $d = 4$ and $a = 5$ | **A2** | **A1** for $d = 4$ or $a = 5$ |
| **9**(b) | $\frac{n}{2}\{2(5)+(n-1)4\}$ soi | **M1** | **FT** *their a* and *their d* |
| | $2n^2+3n-2555\,[*0]$ | **A1** | where * could be = or any inequality sign |
| | Solves *their* 3-term quadratic of the form $ax^2+bx+c$ [*0] by factorising or formula<br>or<br>*their* 3-term quadratic of the form $ax^2+bx*c$ or better if completing the square | **M1** | |
| | 35 | **A1** | |

| | | | |
|---|---|---|---|
| **9**(c) | May work ***consistently*** in $n$ throughout but must conclude in $k$ to earn the final mark | | |
| | $S_{2k}=\frac{2k}{2}\{10+(2k-1)4\}$ soi | **B1** | **FT** *their a* and *their d* |
| | $\frac{2k}{2}\{10+(2k-1)4\}-\frac{k}{2}\{10+(k-1)4\}$ soi | **M1** | **FT** *their a* and *their d* ; condone at most one error |
| | Simplifies as far as e.g.<br>$8k^2 + 6k - (3k + 2k^2)$<br>or $8k^2 + 6k - 3k - 2k^2$ | **A1** | |
| | Correct completion to given answer:<br>$6k^2 + 3k = 3k(1 + 2k)$ | **A1** | |
| | **Alternative method** | | |
| | $\frac{2k}{2}\{2a+(2k-1)d\}$ **and** $a$ = *their* 5 and $d$ = *their* 4 substituted at some point | **(B1)** | |
| | $ak-\frac{d}{2}k+\frac{3}{2}dk^2$ oe | **(M1)** | condone at most one error |
| | $5k-\frac{4}{2}k+\frac{3}{2}\times4\times k^2$ | **(A1)** | |
| | Correct completion to given answer:<br>$6k^2 + 3k = 3k(1 + 2k)$ | **(A1)** | |

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**10** An arithmetic progression, $A$, has first term $a$ and common difference $d$.
The 2nd, 14th and 17th terms of $A$ form the first three terms of a convergent geometric progression, $G$, with common ratio $r$.

**(a)** **(i)** Given that $d \neq 0$, find two expressions for $r$ in terms of $a$ and $d$ and hence show that $a = -17d$. [6]

**(ii)** Find the value of $r$. [2]

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**(b)** The first term of the geometric progression, $G$, is $q$ and the sum to infinity is $\frac{256}{3}$.

Find the sum of the first 20 terms of the **arithmetic** progression, $A$. [7]

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| | | | |
|---|---|---|---|
| **10**(a)(i) | $a + d$, $a + 13d$, $a + 16d$ soi | **B1** | |
| | $\frac{a+13d}{a+d} = \frac{a+16d}{a+13d}$ oe | **M2** | **FT** *their* 3 distinct terms providing of the form<br>$a + kd$ where $k \neq 0$ and at least one is correct<br><br>**M1** for either $[r =]\frac{a+13d}{a+d}$ or<br>$[r =]\frac{a+16d}{a+13d}$ or<br>$[r =]\sqrt{\frac{a+16d}{a+d}}$ |
| | Clears fractions and expands oe:<br>$a^2 + 26ad + 169d^2$<br>$= a^2 + 17ad + 16d^2$ | **A1** | |
| | $9ad + 153d^2 = 0$ or $9ad = -153d^2$ | **A1** | |
| | Convincingly derives $a = -17d$ e.g.<br>$9d(a + 17d) = 0$ [therefore] $a = -17d$ oe | **A1** | |

| | | | |
|---|---|---|---|
| **10**(a)(ii) | $r = 0.25$ oe | **2** | **M1** $\frac{-17d+13d}{-17d+d}$ or $\frac{-17d+16d}{-17d+13d}$<br>or $-16d$ , $-4d$, $-d$ |
| **10**(b) | $\frac{q}{1-0.25} = \frac{256}{3}$ oe | **M1** | **FT** *their* 0.25 providing it is between $-1$ and 1 |
| | $q = 64$ | **A1** | |
| | [$a + d$ = *their* 64 ] $-17d + d$ = *their* 64<br><br>or<br>$a - \frac{a}{17}$ = *their* 64 | **DM1** | **dep** on previous **M1** |
| | $d = -4$ and $a = 68$ oe<br>OR<br>$d = -4$ and $S_{20} = -150d$ oe | **A2** | **A1** for either correct |
| | $S_{20} = \frac{20}{2}\{2(their 68) + 19(their(-4))\}$<br>or<br>$S_{20} = -150(their\ d)$ | **M1** | **FT** *their a* and *d* |
| | 600 | **A1** | |

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