Mark scheme

Simultaneous equations and quadratics

Additional Mathematics 0606 · Questions from Paper 1 and Paper 2

Total marks

60

MARKS FOR EACH QUESTION

1	2	3	4	5	6	7	8	9	10	Total
4	4	5	5	5	6	6	8	7	10	**60**

Question 1 **[4]**

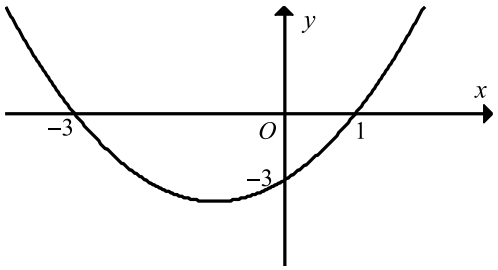
1	$k^2 - 4(2k+5) \quad (< 0)$ $k^2 - 8k - 20 \quad (< 0)$		
	$(k-10)(k+2) \quad (< 0)$	M1	attempt to obtain critical values
	critical values of 10 and –2	A1	correct critical values
	$-2 < k < 10$	A1	correct range
	Alternative 1:		
	$\frac{dy}{dx} = 2(2k+5)x + k$	M1	attempt to differentiate, equate to zero and substitute x value back in to obtain a y value
	When $\frac{dy}{dx} = 0, x = \frac{-k}{2(2k+5)}, y = \frac{8k+20-k^2}{4(2k+5)}$	M1	consider $y = 0$ in order to obtain critical values
	When $y = 0$, obtain critical values of 10 and –2	A1	correct critical values
	$-2 < k < 10$	A1	correct range
	Alternative 2:		
	$y = (2k+5)\left(\left(x + \frac{k}{2(2k+5)}\right)^2 - \frac{k^2}{4(2k+5)}\right) + 1$	M1	attempt to complete the square and consider '$1 - \frac{k^2}{4(2k+5)}$'
	Looking at $1 - \frac{k^2}{4(2k+5)} = 0$ leads to	M1	attempt to solve above = to 0, to obtain critical values
	critical values of 10 and –2	A1	correct critical values
	$-2 < k < 10$	A1	correct range

STEM | Exam Builder

Question 2 [4]

2	$5x^2 - 10x - 15 * 0$ oe	**M1**	where * is any inequality sign or =
	Factorises *their* 3-term quadratic expression or solves *their* 3-term quadratic equation	**M1**	
	Critical values –1 and 3	**A1**	
	$-1 \leqslant x \leqslant 3$	**A1**	Do not accept separate inequalities unless connected with 'and'

Question 3 [5]

3(a)	$(x+1)^2 - 4$ oe $(-1, -4)$	**B3**	**B2** $y = (x+1)^2 - 4$ oe or **B1** $y = (x+1)^2 + k$, where k is a constant
3(b)	Correct curve  	**B2**	**B1** for correct shape in 4 quadrants with minimum in the 3rd quadrant **B1** for an attempt at a quadratic graph with intercepts marked at –3 on the y-axis and –3 and 1 on the x-axis

STEM | Exam Builder

Question 4 **[5]**

4	$x-2\left(4-\sqrt{3}x\right)=5\sqrt{3}$	**M1**	Eliminate y
	$x=\dfrac{5\sqrt{3}+8}{2\sqrt{3}+1}$	**A1**	
	$x=\dfrac{\left(5\sqrt{3}+8\right)\left(2\sqrt{3}-1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}$	**M1**	Multiply by $\left(a\sqrt{b}+c\right)$ as appropriate
	$x=2+\sqrt{3}$	**A1**	
	$y=1-2\sqrt{3}$	**A1**	
	Alternative method $\sqrt{3}\left(5\sqrt{3}+2y\right)+y=4$	**M1**	Eliminate x
	$y=\dfrac{-11}{\left(2\sqrt{3}+1\right)}$	**A1**	
	$y=\dfrac{-11\left(2\sqrt{3}-1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}$	**M1**	Multiply by $\left(a\sqrt{b}+c\right)$ as appropriate
	$y=1-2\sqrt{3}$	**A1**	
	$x=2+\sqrt{3}$	**A1**	

Question 5 **[5]**

5	Either $7^x\times7^{2y}$ or $49^{\frac{x}{2}}\times49^y$ or $5^{5x}\times5^{2y}$ or $25^{\frac{5x}{2}}\times25^y$	**M1**	For expressing the terms on the left hand side of either one of the 2 equations in terms of powers of 7, 49, 5 or 25
	$7^x\times7^{2y}=7^0$ or $49^{\frac{x}{2}}\times49^y=49^0$	**A1**	
	$5^{5x}\times5^{2y}=5^{-2}$ or $25^{\frac{5x}{2}}\times25^y=25^{-1}$	**A1**	
	leading to $x+2y=0$ and $5x+2y=-2$	**M1**	For attempt to solve two linear equations, with integer coefficients and constants, in terms of x and y
	$x=-\dfrac{1}{2},\ y=\dfrac{1}{4}$	**A1**	

STEM | Exam Builder

Question 6 **[6]**

6	$x=\dfrac{2\pm\sqrt{4+4\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}}{2\left(\sqrt{5}-1\right)}$	**M1**	For a correct use of the quadratic formula with sufficient detail
	$x=\dfrac{2\pm2\sqrt{5}}{2\left(\sqrt{5}-1\right)}$ or $x=\dfrac{1\pm\sqrt{5}}{\left(\sqrt{5}-1\right)}$	**2**	**Dep M1** for attempt to simplify to obtain 2 real roots **A1** for either
	$x=\dfrac{\left(\sqrt{5+1}\right)}{\left(\sqrt{5-1}\right)}\times\dfrac{\left(\sqrt{5+1}\right)}{\left(\sqrt{5+1}\right)}$	**M1**	For attempt at rationalisation
	$x=\dfrac{3}{2}+\dfrac{\sqrt{5}}{2}$	**A1**	
	$x=-1$	**B1**	

STEM | Exam Builder

Question 7 **[6]**

7	$4x^2+kx+k-2=2x+1$	**M1**	
	$4x^2+(k-2)x+k-3$ [*0] soi	**A1**	* can be $<,\ >,\ =,\ \leq, \geq$
	$(k-2)^2-4(4)(k-3)$	**M1**	
	$k^2-20k+52*0$	**A1**	
	$k=\dfrac{-(-20)\pm\sqrt{(-20)^2-4(1)(52)}}{2}$	**M1**	
	$k=10\pm\sqrt{48}$ oe isw	**A1**	
	Alternative (using calculus) $2=8x+k$ oe	**(M1)**	
	$y=4x^2+(2-8x)x+2-8x-2$ or $y=-4x^2-6x$	**(M1)**	
	$0=4x^2+8x+1$	**(A1)**	
	$x=\dfrac{-8\pm\sqrt{8^2-4(4)(1)}}{8}$	**(M1)**	
	$x=-1\pm\dfrac{\sqrt{48}}{8}$ oe	**(A1)**	
	for $k=10\pm\sqrt{48}$ oe	**(A1)**	

Question 8 **[8]**

8	**(i)**	$f(x)=3x^2+12x+2=3(x+2)^2-10$ $a=3$ $b=2$ $c=-10$	B1 B1 B1	
	(ii)	minimum $f(x)=-10$ at $x=-2$	B1FT B1FT	
	(iii)	$f\left(\dfrac{1}{y}\right)=0 \rightarrow \left(\dfrac{1}{y}\right)=(\pm)\sqrt{\dfrac{10}{3}}-2$	M1	obtain explicit expression for $\dfrac{1}{y}$ or y
		$y=-5.74,\ -0.26$	A1, A1	

STEM | Exam Builder

Question 9 [7]

9	Eliminates y (or x) and full attempt at expansion	**M1**	
	$4x^2 - 8x - 96 = 0$ or $y^2 + 12y - 64 = 0$	**A1**	
	Factorise 3 term relevant quadratic	**M1**	Or use correct formula
	$x = -4$ and 6 or $y = -16$ and 4	**A1**	
	$y = -16$ and 4 or $x = -4$ and 6	**A1√**	
	Uses Pythagoras for relevant points	**M1**	
	22.4 or $\sqrt{500}$ or $10\sqrt{5}$	**A1**	cao

Question 10 [10]

10	$x^2 - 2x - 3 = 0$ or $y^2 - 6y + 5 = 0$	M1	substitution and simplification to obtain a three term quadratic equation in one variable
	leading to (3, 5) and (–1, 1)	A1,A1	A1 for each 'pair' from a correct quadratic equation, correctly obtained.
	Midpoint (1, 3)	B1cao	midpoint
	(Gradient – 1) Perpendicular bisector $y = 4 - x$	M1	perpendicular bisector, must be using *their* perpendicular gradient and *their* midpoint
	Meets the curve again if $x^2 + 10x - 15 = 0$ or $y^2 - 18y + 41 = 0$	M1	substitution and simplification to obtain a three term quadratic equation in one variable.
	leading to $x = -5 \pm 2\sqrt{10}$, $y = 9 \mp 2\sqrt{10}$	A1,A1	A1 for each 'pair'
	$CD^2 = \left(4\sqrt{10}\right)^2 + \left(4\sqrt{10}\right)^2$	M1	Pythagoras using *their* coordinates from solution of second quadratic. $(x_1 - x_2)^2 + (y_1 - y_2)^2$ must be seen if not using correct coordinates.
	$CD = 8\sqrt{5}$	A1	A1 for $8\sqrt{5}$ from $\sqrt{320}$ and all correct so far.

STEM Exam Builder