Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

STEM | Exam Builder

Simultaneous equations and quadratics

Time allowed: 1 hour 30 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 1-2, 5-6, 10 (Paper 1): no calculator.
Questions 3-4, 7-9 (Paper 2): calculator allowed.

Total marks

/ 60

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 60 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

STEM | Exam Builder

Print at 100% (Actual size) so diagrams keep their real measurements.

1 Given that the graph of $y = (2k + 5)x^2 + kx + 1$ does not meet the x-axis, find the possible values of k. [4]

The discriminant: nature of roots, line and curve Click for more variations

STEM | Exam Builder

1	$k^2-4(2k+5)\quad(<0)$ $k^2-8k-20\quad(<0)$	M1	use of b^2-4ac, (not as part of quadratic formula unless isolated at a later stage) with correct values for a, b and c Do not need to see < at this point
	$(k-10)(k+2)\quad(<0)$	M1	attempt to obtain critical values
	critical values of 10 and –2	A1	correct critical values
	$-2<k<10$	A1	correct range
	Alternative 1:		
	$\frac{dy}{dx}=2(2k+5)x+k$	M1	attempt to differentiate, equate to zero and substitute x value back in to obtain a y value
	When $\frac{dy}{dx}=0, x=\frac{-k}{2(2k+5)}, y=\frac{8k+20-k^2}{4(2k+5)}$	M1	consider $y=0$ in order to obtain critical values
	When $y=0$, obtain critical values of 10 and –2	A1	correct critical values
	$-2<k<10$	A1	correct range
	Alternative 2:		
	$y=(2k+5)\left(\left(x+\frac{k}{2(2k+5)}\right)^2-\frac{k^2}{4(2k+5)}\right)+1$	M1	attempt to complete the square and consider $'1-\frac{k^2}{4(2k+5)}'$
	Looking at $1-\frac{k^2}{4(2k+5)}=0$ leads to	M1	attempt to solve above = to 0, to obtain critical values
	critical values of 10 and –2	A1	correct critical values
	$-2<k<10$	A1	correct range

STEM | Exam Builder

2 Solve the inequality $(3-x)(5x+8) \geqslant 9-3x$. [4]

STEM | Exam Builder

Quadratic equations and inequalities Click for more variations

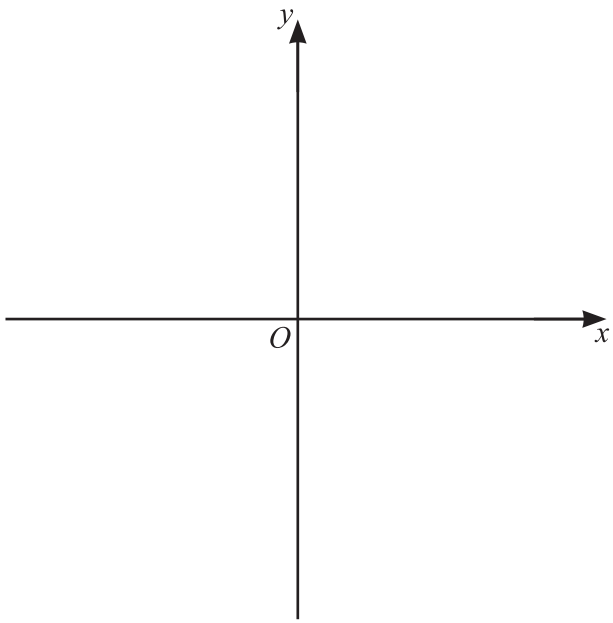
2	$5x^2 - 10x - 15 * 0$ oe	**M1**	where * is any inequality sign or =
	Factorises *their* 3-term quadratic expression or solves *their* 3-term quadratic equation	**M1**	
	Critical values –1 and 3	**A1**	
	$-1 \leqslant x \leqslant 3$	**A1**	Do not accept separate inequalities unless connected with 'and'

STEM | Exam Builder

3 A curve has equation $y = x^2 + 2x - 3$.

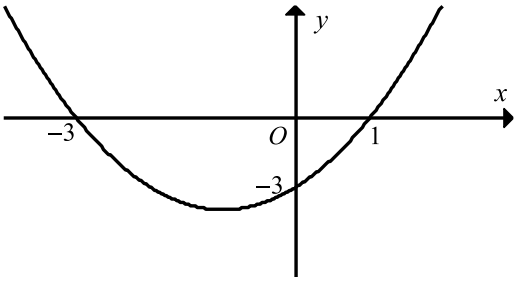
(a) Use the method of completing the square to find the coordinates of the stationary point on the curve. [3]

(b) On the axes, sketch the curve, stating the intercepts with the coordinate axes. [2]



STEM | Exam Builder

Completing the square and quadratic graphs Click for more variations

3(a)	$(x+1)^2-4$ oe $(-1, -4)$	**B3**	**B2** $y=(x+1)^2-4$ oe or **B1** $y=(x+1)^2+k$, where k is a constant
3(b)	Correct curve  	**B2**	**B1** for correct shape in 4 quadrants with minimum in the 3rd quadrant **B1** for an attempt at a quadratic graph with intercepts marked at –3 on the y-axis and –3 and 1 on the x-axis

STEM | Exam Builder

4 Solve the following simultaneous equations for x and y, giving each answer in its simplest surd form.

$$\sqrt{3}x + y = 4$$

$$x - 2y = 5\sqrt{3}$$ [5]

STEM | Exam Builder

Simultaneous equations Click for more variations

4	$x-2\left(4-\sqrt{3}x\right)=5\sqrt{3}$	**M1**	Eliminate y
	$x=\dfrac{5\sqrt{3}+8}{2\sqrt{3}+1}$	**A1**	
	$x=\dfrac{\left(5\sqrt{3}+8\right)\left(2\sqrt{3}-1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}$	**M1**	Multiply by $\left(a\sqrt{b}+c\right)$ as appropriate
	$x=2+\sqrt{3}$	**A1**	
	$y=1-2\sqrt{3}$	**A1**	
	Alternative method $\sqrt{3}\left(5\sqrt{3}+2y\right)+y=4$	**M1**	Eliminate x
	$y=\dfrac{-11}{\left(2\sqrt{3}+1\right)}$	**A1**	
	$y=\dfrac{-11\left(2\sqrt{3}-1\right)}{\left(2\sqrt{3}+1\right)\left(2\sqrt{3}-1\right)}$	**M1**	Multiply by $\left(a\sqrt{b}+c\right)$ as appropriate
	$y=1-2\sqrt{3}$	**A1**	
	$x=2+\sqrt{3}$	**A1**	

STEM | Exam Builder

5 Given that $7^x \times 49^y = 1$ and $5^{5x} \times 125^{\frac{2y}{3}} = \frac{1}{25}$, calculate the value of x and of y. [5]

STEM | Exam Builder

Simultaneous equations Click for more variations

5	Either $7^x \times 7^{2y}$ or $49^{\frac{x}{2}} \times 49^y$ or $5^{5x} \times 5^{2y}$ or $25^{\frac{5x}{2}} \times 25^y$	**M1**	For expressing the terms on the left hand side of either one of the 2 equations in terms of powers of 7, 49, 5 or 25
	$7^x \times 7^{2y} = 7^0$ or $49^{\frac{x}{2}} \times 49^y = 49^0$	**A1**	
	$5^{5x} \times 5^{2y} = 5^{-2}$ or $25^{\frac{5x}{2}} \times 25^y = 25^{-1}$	**A1**	
	leading to $x + 2y = 0$ and $5x + 2y = -2$	**M1**	For attempt to solve two linear equations, with integer coefficients and constants, in terms of x and y
	$x = -\frac{1}{2}, \ y = \frac{1}{4}$	**A1**	

STEM | Exam Builder

6 DO NOT USE A CALCULATOR IN THIS QUESTION.

Solve the equation $\left(\sqrt{5}-1\right)x^2-2x-\left(\sqrt{5}+1\right)=0$, giving your answers in the form $a+b\sqrt{5}$, where a and b are constants. [6]

STEM | Exam Builder

Indices and surds Click for more variations

6	$x=\dfrac{2\pm\sqrt{4+4\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}}{2\left(\sqrt{5}-1\right)}$	**M1**	For a correct use of the quadratic formula with sufficient detail
	$x=\dfrac{2\pm2\sqrt{5}}{2\left(\sqrt{5}-1\right)}$ or $x=\dfrac{1\pm\sqrt{5}}{\left(\sqrt{5}-1\right)}$	**2**	**Dep M1** for attempt to simplify to obtain 2 real roots **A1** for either
	$x=\dfrac{\left(\sqrt{5+1}\right)}{\left(\sqrt{5-1}\right)}\times\dfrac{\left(\sqrt{5+1}\right)}{\left(\sqrt{5+1}\right)}$	**M1**	For attempt at rationalisation
	$x=\dfrac{3}{2}+\dfrac{\sqrt{5}}{2}$	**A1**	
	$x=-1$	**B1**	

STEM | Exam Builder

7 Find the exact values of the constant k for which the line $y = 2x + 1$ is a tangent to the curve $y = 4x^2 + kx + k - 2$. [6]

STEM | Exam Builder

The discriminant: nature of roots, line and curve Click for more variations

7	$4x^2 + kx + k - 2 = 2x + 1$	**M1**	
	$4x^2 + (k-2)x + k - 3$ [*0] soi	**A1**	* can be <, >, =, ≤, ≥
	$(k-2)^2 - 4(4)(k-3)$	**M1**	
	$k^2 - 20k + 52 * 0$	**A1**	
	$k = \dfrac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(52)}}{2}$	**M1**	
	$k = 10 \pm \sqrt{48}$ oe isw	**A1**	
	Alternative (using calculus) $2 = 8x + k$ oe	**(M1)**	
	$y = 4x^2 + (2-8x)x + 2 - 8x - 2$ or $y = -4x^2 - 6x$	**(M1)**	
	$0 = 4x^2 + 8x + 1$	**(A1)**	
	$x = \dfrac{-8 \pm \sqrt{8^2 - 4(4)(1)}}{8}$	**(M1)**	
	$x = -1 \pm \dfrac{\sqrt{48}}{8}$ oe	**(A1)**	
	for $k = 10 \pm \sqrt{48}$ oe	**(A1)**	

STEM | Exam Builder

8 Given that $f(x) = 3x^2 + 12x + 2$,

(i) find values of a, b and c such that $f(x) = a(x + b)^2 + c$, [3]

(ii) state the minimum value of $f(x)$ and the value of x at which it occurs, [2]

(iii) solve $f\left(\frac{1}{y}\right) = 0$, giving each answer for y correct to 2 decimal places. [3]

STEM | Exam Builder

Completing the square and quadratic graphs Click for more variations

8	**(i)**	$f(x) = 3x^2 + 12x + 2 = 3(x+2)^2 - 10$ $a = 3$ $b = 2$ $c = -10$	 B1 B1 B1	
	(ii)	minimum $f(x) = -10$ at $x = -2$	B1FT B1FT	
	(iii)	$f\left(\frac{1}{y}\right) = 0 \rightarrow \left(\frac{1}{y}\right) = (\pm)\sqrt{\frac{10}{3}} - 2$	M1	obtain explicit expression for $\frac{1}{y}$ or y
		$y = -5.74, \quad -0.26$	A1, A1	

STEM | Exam Builder

9 The line $y = 2x - 8$ cuts the curve $2x^2 + y^2 - 5xy + 32 = 0$ at the points A and B. Find the length of the line AB. [7]

Simultaneous equations Click for more variations

STEM | Exam Builder

9	Eliminates y (or x) and full attempt at expansion	**M1**	
	$4x^2 - 8x - 96 = 0$ or $y^2 + 12y - 64 = 0$	**A1**	
	Factorise 3 term relevant quadratic	**M1**	Or use correct formula
	$x = -4$ and 6 or $y = -16$ and 4	**A1**	
	$y = -16$ and 4 or $x = -4$ and 6	**A1√**	
	Uses Pythagoras for relevant points	**M1**	
	22.4 or $\sqrt{500}$ or $10\sqrt{5}$	**A1**	cao

STEM | Exam Builder

10 The line $x - y + 2 = 0$ intersects the curve $2x^2 - y^2 + 2x + 1 = 0$ at the points A and B. The perpendicular bisector of the line AB intersects the curve at the points C and D. Find the length of the line CD in the form $a\sqrt{5}$, where a is an integer. [10]

STEM | Exam Builder

Simultaneous equations Click for more variations

10	$x^2 - 2x - 3 = 0$ or $y^2 - 6y + 5 = 0$	M1	substitution and simplification to obtain a three term quadratic equation in one variable
	leading to (3, 5) and (–1, 1)	A1,A1	A1 for each 'pair' from a correct quadratic equation, correctly obtained.
	Midpoint (1, 3)	B1cao	midpoint
	(Gradient – 1) Perpendicular bisector $y = 4 - x$	M1	perpendicular bisector, must be using *their* perpendicular gradient and *their* midpoint
	Meets the curve again if $x^2 + 10x - 15 = 0$ or $y^2 - 18y + 41 = 0$	M1	substitution and simplification to obtain a three term quadratic equation in one variable.
	leading to $x = -5 \pm 2\sqrt{10}$, $y = 9 \mp 2\sqrt{10}$	A1,A1	A1 for each 'pair'
	$CD^2 = \left(4\sqrt{10}\right)^2 + \left(4\sqrt{10}\right)^2$	M1	Pythagoras using *their* coordinates from solution of second quadratic. $(x_1 - x_2)^2 + (y_1 - y_2)^2$ must be seen if not using correct coordinates.
	$CD = 8\sqrt{5}$	A1	A1 for $8\sqrt{5}$ from $\sqrt{320}$ and all correct so far.

STEM | Exam Builder