Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Solving logarithmic equations

Time allowed: 1 hour 40 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 1, 3-4, 9 (Paper 1): no calculator.
Questions 2, 5-8, 10 (Paper 2): calculator allowed.

Total marks

/ 64

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 64 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 Given that $\log_3 r + 2\log_9 s = 8$, find the value of rs. [3]

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1	$rs = 6561$ isw	**3**	**B2** for $\log_3 rs = 8$ or $\log_3 (rs)^2 = 16$ **B1** for $2\log_9 s = \frac{2\log_3 s}{\log_3 9}$ soi
	Alternative 1		
	$rs = 6561$ isw	**(3)**	**B2** for $\log_9 rs = 4$ or $\log_9 (rs)^2 = 8$ **B1** for $\log_3 r = \frac{\log_9 r}{\log_9 3}$ soi
	Alternative 2		
	$rs = 6561$ isw	**(3)**	**B2** for $\log rs = 8\log 3$ or $\log rs = 4\log 9$ for any base **B1** for $\frac{\log r}{\log 3} + \frac{2\log s}{\log 9} = 8$ soi for any base

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2 Solve the equation $64^{x+\frac{1}{3}} + 2^{3x} - 3 = 0$. [4]

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2	$4\times 8^{2x}+8^{x}-3[=0]$ or $4\times(2^{3x})^{2}+2^{3x}-3[=0]$ oe	**B1**	
	$\left(4(8^{x})-3\right)\left(8^{x}+1\right)$ or $\left(4(2^{3x})-3\right)\left(2^{3x}+1\right)$ oe	**M1**	**FT** *their* 3-term quadratic in 8^{x} or 2^{3x}
	$8^{x}=\frac{3}{4}$ or $2^{3x}=\frac{3}{4}$ oe, nfww	**A1**	
	$x=\log_{8}(0.75)$ or $\frac{1}{3}\log_{2}(0.75)$ or –0.138[34…] oe	**A1**	and no other solutions

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3 Solve the equation $\log_2(x+1) - 4\log_{(x+1)}2 = 3$. [5]

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3	Correct change of base e.g. $\log_2(x+1)-\dfrac{4\log_2 2}{\log_2(x+1)}=3$ or $\dfrac{\log_{(x+1)}(x+1)}{\log_{(x+1)}2}-4\log_{(x+1)}2=3$	**B1**	
	$\left(\log_2(x+1)\right)^2-3\log_2(x+1)-4 \quad [=0]$ oe or $4\left(\log_{(x+1)}2\right)^2+3\log_{(x+1)}2-1 \quad [=0]$ oe	**B1**	
	Factorises or solves e.g. $\left(\log_2(x+1)-4\right)\left(\log_2(x+1)+1\right)=0$ or $\left(4\log_{(x+1)}2-1\right)\left(\log_{(x+1)}2+1\right)=0$	**M1**	**FT** *their* 3-term quadratic in a suitable logarithm providing correct change of base seen
	$\log_2(x+1)=their 4$ and $\log_2(x+1)=their(-1)$ or $\log_{(x+1)}2=their\dfrac{1}{4}$ and $\log_{(x+1)}2=their(-1)$ **and** correctly solves as far as x = … at least once	**M1**	**dep** on previous **M1** **FT** $\log_2(x+1)=a$ or $\log_{(x+1)}2=b$ where a and b are constants
	$x=15,\ x=-\dfrac{1}{2}$ nfww mark final answer	**A1**	

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4 Find the x-coordinates of the points of intersection of the following curves.

$$y = 4\ln x \quad \text{and} \quad y = 5 - \frac{3}{\ln\left(x^2\right)}$$

Give your answers in exact form. [5]

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4	$8(\ln x)^2 - 10\ln x + 3 \quad (=0)$	**2**	**M1** for elimination of y and re-arrangement to a 3 term quadratic in terms of $\ln x$
	$\ln x = 0.75, \quad \ln x = 0.5$	**2**	**Dep** on previous **M** mark for correct method of solution
	$x = e^{0.75}, \quad x = e^{0.5}$	**A1**	For both

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5 Do not use a calculator in this question.

(i) Find the value of $-\log_p p^2$. [1]

(ii) Find $\lg\left(\frac{1}{10^n}\right)$. [1]

(iii) Show that $\frac{\lg 20 - \lg 4}{\log_5 10} = (\lg y)^2$, where y is a constant to be found. [2]

(iv) Solve $\log_r 2x + \log_r 3x = \log_r 600$. [2]

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5	**(i)**	-2	**B1**	
	(ii)	$-n$	**B1**	
	(iii)	$\frac{\lg 5}{\log_5 10} = [(\lg y)^2]$ or $\frac{\lg 20 - \lg 4}{1 \diagup \lg 5} = [(\lg y)^2]$	**M1**	One log law used correctly
		correct completion to $(\lg 5)^2$ isw	**A1**	answer only does not score
	(iv)	$[\log_r] 6x^2 = [\log_r] 600$	**B1**	Condone base missing
		$x = 10$ only	**B1**	

6 **(a)** Write $2\lg x - (\lg(x+6) + \lg 3)$ as a single logarithm to base 10. [2]

(b) Hence solve the equation $2\lg x - (\lg(x+6) + \lg 3) = 0$. [4]

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6(a)	$\lg\frac{x^2}{3(x+6)}$ oe, nfww	**B2**	**B1** for any two log laws applied correctly e.g. $\lg\frac{x^2}{x+6}-\lg 3$
6(b)	$\lg\frac{x^2}{3(x+6)}=\lg 1$ or $10^0=\frac{x^2}{3(x+6)}$	**B1**	FT *their* $\lg\frac{x^2}{3(x+6)}$ providing a single logarithm
	$x^2-3x-18=0$	**B1**	**dep** on **B2** in **part (a)**
	Factorises or solves their 3-term quadratic	**M1**	
	$x = 6$ indicated as only solution	**A1**	**dep** on **B2** in **part (a)**

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7 **(a)** Solve the equation $\log_6(2x-3)=\frac{1}{2}$. Give your answer in exact form. [2]

(b) Solve the equation $\ln 2u-\ln(u-4)=1$. Give your answer in exact form. [3]

(c) Solve the equation $\frac{3^v}{27^{2v-5}}=9$. [3]

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7(a)	$2x-3=6^{\frac{1}{2}}$ oe, soi	**M1**	
	$x=\dfrac{6^{\frac{1}{2}}+3}{2}$ or $x=\dfrac{\sqrt{6}+3}{2}$	**A1**	
7(b)	$\ln\dfrac{2u}{u-4}=\ln \mathrm{e}$ soi or $\ln\dfrac{2u}{u-4}=1$ soi or $\ln 2u=\ln \mathrm{e}(u-4)$ soi	**M1**	Condone one sign or bracketing error
	$\dfrac{2u}{u-4}=\mathrm{e}$ or $2u=\mathrm{e}(u-4)$ oe	**M1**	**FT** *their* logarithmic equation
	$u=\dfrac{4\mathrm{e}}{\mathrm{e}-2}$ or $u=\dfrac{-4\mathrm{e}}{2-\mathrm{e}}$ or equivalent exact form	**A1**	
7(c)	$\dfrac{3^v}{\left(3^3\right)^{2v-5}}=3^2$ oe soi or $\dfrac{9^{\frac{v}{2}}}{\left(9^{\frac{3}{2}}\right)^{2v-5}}=9$ oe soi or $\log 3^v-\log 27^{2v-5}=\log 9$ oe soi	**B1**	
	$15-5v=2$ oe or $v\log 3-(2v-5)\log 27=\log 9$	**M1**	**FT** *their* exponential equation in the same base or *their* logarithmic equation with any consistent base, providing *their* exponential or logarithmic equation has at most one sign or arithmetic error
	$v=\dfrac{13}{5}$ oe	**A1**	

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8 **(a)** Solve the equation $\dfrac{9^{5x}}{27^{x-2}} = 243$. [3]

(b) $\log_a \sqrt{b} - \dfrac{1}{2} = \log_b a$, where $a > 0$ and $b > 0$.

Solve this equation for b, giving your answers in terms of a. [5]

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8(a)	$\frac{3^{10x}}{3^{3x-6}}[=243]$ oe or $\log 9^{5x} - \log 27^{x-2} = \log 243$ oe	**B1**	
	$3^{7x+6} = 3^5$ soi oe or $5x(\log 9) - (x-2)\log 27 = \log 243$	**M1**	
	$x = -\frac{1}{7}$	**A1**	

8(b)	$\frac{1}{2}\log_a b - \frac{1}{2} = \frac{1}{\log_a b}$ or $\frac{\frac{1}{2}}{\log_b a} - \frac{1}{2} = \log_b a$	**B2**	**B1** for bringing down the power of $\frac{1}{2}$ e.g. $\frac{1}{2}\log_a b$ or for a change of base e.g. $\frac{1}{\log_a b}$
	Clears the fraction and rearranges $\frac{1}{2}(\log_a b)^2 - \frac{1}{2}\log_a b = 1$ oe $(\log_a b)^2 - \log_a b - 2 = 0$ oe or let $x = \log_a b$ $x^2 - x - 2 = 0$ oe or $\frac{1}{2} - \frac{1}{2}\log_b a = (\log_b a)^2$ $0 = 2(\log_b a)^2 + \log_b a - 1$ oe or let $y = \log_b a$ $2y^2 + y - 1 = 0$	**M1**	
	$(\log_a b - 2)(\log_a b + 1)$ oe or $(2\log_b a - 1)(\log_b a + 1)$	**M1**	
	[$\log_a b = 2$, $\log_a b = -1$ or $\log_b a = \frac{1}{2}$, $\log_b a = -1$ leading to] $b = a^2$, $b =$ oe	**A1**	

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9 **(a)** Express $2+3\lg x-\lg y$ as a single logarithm to base 10. [3]

(b) **(i)** Solve $6x+7-\frac{3}{x}=0$. [2]

(ii) Hence, given that $6\log_a 3+7-3\log_3 a=0$, find the possible values of a. [4]

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9(a)	$\lg 100 = 2$	**B1**	
	$3\lg x = \lg x^3$	**B1**	
	$\lg \frac{100x^3}{y}$	**B1**	
9(b)(i)	$6x^2 + 7x - 3 = 0$ $(2x+3)(3x-1) = 0$	**M1**	For obtaining in suitable quadratic form and attempt to solve
	$x = -\frac{3}{2}$ $x = \frac{1}{3}$	**A1**	For both

9(b)(ii)	$x = \log_a 3$ $\frac{1}{3} = \log_a 3$	**M1**	For realising connection with part **(i)** and attempt to solve $\frac{1}{3} = \log_a 3$ or $-\frac{3}{2} = \log_a 3$
	$a = 27$	**A1**	
	$-\frac{3}{2} = \log_a 3$	**M1**	For realising connection with part **(i)** and attempt to solve $-\frac{3}{2} = \log_a 3$ or $\frac{1}{3} = \log_a 3$
	$a = \left(\frac{1}{3}\right)^{\frac{2}{3}}$ or 0.481or $\left(\frac{1}{9}\right)^{\frac{1}{3}}$ oe	**A1**	

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10 **(a)** **(i)** Solve $\frac{5^{2x+3}}{25^{2x}} = \frac{25^{2-x}}{125^{x}}$. [3]

(ii) Solve $1g\ y + 1g(y - 15) = 2$. [4]

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(b) Without using a calculator, and showing each stage of your working, find the value of

$2\log_{12} 4 - \frac{1}{2}\log_{12} 81 + 4\log_{12} 3$. [3]

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10	**(a)(i)** Express in powers of 5. $\left(\frac{5^{2x+3}}{5^{4x}} = \frac{5^{2(2-x)}}{5^{3x}}\right)$	M1
	Use rules of indices $(2x + 3 - 4x = 2(2 - x) - 3x)$	M1
	$\frac{1}{3}$	A1
	(ii) LHS = $\lg y(y - 15)$	B1
	$2 = \lg 100$	B1
	Solve 3 term quadratic	M1
	20 only	A1
	(b) $\log_{12} 16 - \log_{12} 9 + \log_{12} 81$	B1
	Correctly combine 3 logarithms	M1
	2	A1
		[10]

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