Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Stationary points

Time allowed: 1 hour 50 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 1-2, 4, 7-10 (Paper 1): no calculator.
Questions 3, 5-6 (Paper 2): calculator allowed.

Total marks

/ 72

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 72 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 Find the coordinates of the stationary point on the curve $y = x^2 + \frac{16}{x}$. [4]

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1	$\frac{dy}{dx} = 2x - \frac{16}{x^2}$	**M1** **A1**	for attempt to differentiate all correct
	When $\frac{dy}{dx} = 0$,	**DM1**	for equating $\frac{dy}{dx}$ to zero and an attempt to solve for x.
	$x = 2, y = 12$	**A1**	**A1** for both, but no extra solutions

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2 Find the coordinates of the stationary point of the curve $y = \dfrac{x+2}{\sqrt{2x-1}}$. [6]

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2	attempt to differentiate quotient or equivalent product	**M1**	
	$\frac{d}{dx}(2x-1)^{\frac{1}{2}} = (2x-1)^{-\frac{1}{2}}$ for a quotient $\frac{d}{dx}(2x-1)^{-\frac{1}{2}} = -(2x-1)^{-\frac{3}{2}}$ for a product	**B1**	
	either $\frac{dy}{dx} = \frac{\sqrt{2x-1} - (x+2)\left[(2x-1)^{-\frac{1}{2}}\right]}{\left(\sqrt{2x-1}\right)^2}$ or $\frac{dy}{dx} = (2x-1)^{-\frac{1}{2}} - (x+2)\left[(2x-1)^{-\frac{3}{2}}\right]$	**A1**	All other terms correct
	When $\frac{dy}{dx} = 0$, $2x-1 = x+2$	**M1**	equate to zero and attempt to solve
	$x = 3$	**A1**	
	$y = \sqrt{5}$, $\frac{5}{\sqrt{5}}$, 2.24	**A1**	

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3 Show that the curve $y = x - \ln(x^2 + 2x)$ has exactly one stationary point.

Find the x-coordinate of this point. [6]

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3	$\frac{dy}{dx} = 1 - \frac{2x+2}{x^2+2x}$	**2**	**B1** for $\frac{d(-\ln(x^2+2x))}{dx} = \frac{1}{x^2+2x} \times f(x)$ soi
	Equates *their* first derivative to 0 and simplifies as far as $2(x + 1) = x(x+ 2)$ oe	**M1**	**FT** *their* $\frac{dy}{dx}$
	$x^2 - 2 = 0$ or $x^2 = 2$	**A1**	
	$x = \sqrt{2}$ $[x = -\sqrt{2}]$	**A1**	
	Justification that the negative solution should be rejected e.g. $x = -\sqrt{2}$ results in $y = 2 - \ln(2 - 2\sqrt{2})$ which is impossible	**B1**	

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4 A curve has equation $y = 6\cos\frac{x}{2} + 4\sin\frac{x}{2}$, for $0 < x < 2\pi$ radians.

(i) Find the x-coordinate of the stationary point on the curve. [5]

(ii) Determine the nature of this stationary point. [2]

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4	**(i)** $\frac{dy}{dx} = -3\sin\frac{x}{2} + 2\cos\frac{x}{2}$	M1 A1, A1	M1 for attempt to differentiate A1 for each correct term
	When $\frac{dy}{dx} = 0$, $\tan\frac{x}{2} = \frac{2}{3}$	M1	M1 for their $\frac{dy}{dx} = 0$
	$x = 1.18$	A1	A1 correct solution only
	(ii) $\frac{d^2y}{dx^2} = -\frac{3}{2}\cos\frac{x}{2} - \sin\frac{x}{2}$	M1	M1 for a valid method – needs to be seen,
	When $x = 1.18$ $\frac{d^2y}{dx^2}$ is –ve (–1.8)		
	Maximum	A1 [7]	

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5 A curve has equation $y = 32x^2 + \frac{1}{8x^2}$ where $x \neq 0$.

(a) Find the coordinates of the stationary points of the curve. [5]

(b) These stationary points have the same nature. Use the second derivative test to determine whether they are maximum points or minimum points. [3]

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5(a)	$\frac{dy}{dx} = 64x - \frac{2x^{-3}}{8}$ oe, isw	**B2**	**B1** for $\frac{dy}{dx} = 64x + kx^{-3}$ or $\frac{dy}{dx} = kx - \frac{2x^{-3}}{8}$ where k is a non-zero constant or **SC1** for $\frac{dy}{dx} = 64x - \frac{2}{8}x^{-3} + c$
	their $\frac{dy}{dx} = 0$ and attempt to solve	**M1**	**FT** *their* derivative providing it has two terms and at least one term is a correct power of x
	(0.25, 4), (–0.25, 4) nfww, isw	**A2**	**A1** for either stationary point correct or for $x = \pm 0.25$ nfww or, if $\frac{dy}{dx} = 64x - 16x^{-3}$, then award **SC2** for $\left(\pm\frac{1}{\sqrt{2}}, \frac{65}{4}\right)$ oe or **SC1** for either of these stationary points or $x = \pm\frac{1}{\sqrt{2}}$ oe

5(b)	Correct second derivative: $\frac{d^2y}{dx^2} = 64 + \frac{3}{4}x^{-4}$ oe, isw	**M1**	**FT** *their* $\frac{dy}{dx} = mx + nx^{-3}$ where $m \neq 0$ and $n \neq 0$ seen in part (a)
	$64 + \frac{3}{4}\left(\frac{1}{4}\right)^{-4} = 256$ or $64 + \frac{3}{4}\left(\frac{1}{4}\right)^{-4} > 0$ or when $x = 0.25$ $\frac{d^2y}{dx^2} = 256$ or $\frac{d^2y}{dx^2} > 0$ oe **and** minimum [points] oe OR $64 + \frac{3}{4}\left(-\frac{1}{4}\right)^{-4} = 256$ or $64 + \frac{3}{4}\left(-\frac{1}{4}\right)^{-4} > 0$ or when $x = -0.25$ $\frac{d^2y}{dx^2} = 256$ or $\frac{d^2y}{dx^2} > 0$ oe **and** minimum [points] oe OR $\frac{d^2y}{dx^2} = 64 + \frac{3}{4x^4}$ and this is positive for any value of x **and** minimum [points]	**A2**	**dep** on $x = \pm 0.25$ nfww in part **(a)** **A1 dep** on $x = 0.25$ or $x = -0.25$ nfww in part (a) for correctly showing or stating $\frac{d^2y}{dx^2}\left[= 64 + \frac{3}{4x^4}\right]$ is positive

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6 **(a)** Find the x-coordinates of the stationary points on the curve $y = 3\ln x + x^2 - 7x$, where $x > 0$. [5]

(b) Determine the nature of each of these stationary points. [3]

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6(a)	$\frac{dy}{dx} = \frac{3}{x} + 2x - 7$	**B2**	**B1** for the first term correct and one other term correct or for all terms correct with extra terms seen
	Equates *their* $\frac{dy}{dx}$ to zero and rearranges to 3-term quadratic in x	**M1**	
	Solves *their* 3-term quadratic	**M1**	**Dep** on previous M1
	$x = 0.5,\ 3$ nfww isw	**A1**	no extra solutions
6(b)	$\frac{d^2y}{dx^2} = -\frac{3}{x^2} + 2$	**M1**	**FT** *their* $\frac{dy}{dx}$ providing B1 earned in **(a)**
	$x = 0.5$, $\frac{d^2y}{dx^2} < 0 \rightarrow \max$ or $\frac{d^2y}{dx^2} = -10 \rightarrow \max$	**A1**	
	$x = 3$, $\frac{d^2y}{dx^2} > 0 \rightarrow \min$ or $\frac{d^2y}{dx^2} = \frac{5}{3} \rightarrow \min$	**A1**	
	Alternative method		
	Considers gradient at $x - h$ and $x + h$ for $x = 0.5$ or $x = 3$ [where h is small] or Considers y-values at $x - h$ and $x + h$ for $x = 0.5$ or $x = 3$ [where h is small]	**(M1)**	**FT** *their* $\frac{dy}{dx}$ providing B1 earned in **(a)**
	Correct conclusion for one turning point max at $x = 0.5$ or min at $x = 3$	**(A1)**	
	Correct method and conclusion for second turning point	**(A1)**	

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7 Show that the curve $y = (3x^2 + 8)^{\frac{5}{3}}$ has only one stationary point. Find the coordinates of this stationary point and determine its nature. [8]

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7	differentiation to obtain answer in the form $p\left(3x^2+8\right)^{\frac{2}{3}}$ or $qx\left(3x^2+8\right)^{\frac{2}{3}}$	**M1**	
	$6x\left(3x^2+8\right)^{\frac{2}{3}}$	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5}{3}\times 6x\left(3x^2+8\right)^{\frac{2}{3}}$	**A1**	all correct
	When $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ only solution is $x=0$	**DM1**	$qx\left(3x^2+8\right)^{\frac{2}{3}}=0$ and attempt to solve
	$x=0$ and $3x^2+8=0$ has no solutions	**A1**	
	Stationary point at $(0,32)$	**A1**	
	correct gradient method with substitution of x values either side of zero or equivalent valid method	**M1**	
	correct conclusion from correct work using a correct $\frac{\mathrm{d}y}{\mathrm{d}x}$	**A1**	

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8 A curve has equation $y = \dfrac{(3x^2 - 5)^{\frac{1}{3}}}{x + 4}$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ can be written in the form $\dfrac{Ax^2 + Bx + C}{(3x^2 - 5)^{\frac{2}{3}}(x + 4)^2}$, where A, B and C are integers. [5]

(b) Hence find the x-coordinates of the stationary points on the curve. Give your answers in their simplest exact form. [3]

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8(a)	$\dfrac{\frac{1}{3}(6x)\left(3x^2-5\right)^{-\frac{2}{3}}(x+4)-\left(3x^2-5\right)^{\frac{1}{3}}}{(x+4)^2}$ or $\frac{1}{3}(6x)\left(3x^2-5\right)^{-\frac{2}{3}}(x+4)^{-1}$ $+\left(3x^2-5\right)^{\frac{1}{3}}\left(-(x+4)^{-2}\right)$	**3**	**B1** for $\frac{1}{3}(6x)\left(3x^2-5\right)^{-\frac{2}{3}}$ soi **M1** for attempt at differentiation of a quotient or valid product **A1** for all other terms correct
	$\left(2x(x+4)-\left(3x^2-5\right)\right)$	**M1**	**M1 dep** on previous **M** mark for simplification of *their* numerator Allow one sign slip but must be dealing with two quadratic terms.
	$\dfrac{-x^2+8x+5}{\left(3x^2-5\right)^{\frac{2}{3}}(x+4)^2}$	**A1**	

8(b)	$-x^2+8x+5=0$ oe, together with attempt to solve	**M1**	Allow for use of *their* quadratic numerator from part (a)
	$4\pm\sqrt{21}$	**2**	**A2** for both **A1** for $\frac{8+\sqrt{84}}{2}$ or $\frac{8-\sqrt{84}}{2}$ oe

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9 A curve has equation $y = \left(\dfrac{x^2-1}{x^2+1}\right)^4$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ can be written as $\dfrac{Ax\left(x^2-1\right)^3}{\left(x^2+1\right)^5}$, where A is a positive integer to be found. [5]

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(b) **(i)** Show that the curve has stationary points where $x = -1$, $x = 0$ and $x = 1$. [1]

(ii) Use the first derivative test to determine which two stationary points have the same nature and state whether they are maximum or minimum points. [2]

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9(a)	Correct derivative $\frac{d}{dx}\left(\frac{x^2-1}{x^2+1}\right)=\frac{(x^2+1)(2x)-(x^2-1)(2x)}{(x^2+1)^2}$ or $2x\left(x^2+1\right)^{-1}-2x\left(x^2-1\right)\left(x^2+1\right)^{-2}$	**2**	**M1** for an attempt at the quotient or product rule **A1** fully correct
	Correct derivative $\frac{dy}{dx}=4\left(\frac{x^2-1}{x^2+1}\right)^3\left(\frac{(x^2+1)(2x)-(x^2-1)(2x)}{(x^2+1)^2}\right)$ or $4\left(\frac{x^2-1}{x^2+1}\right)^3\left(2x\left(x^2+1\right)^{-1}-2x\left(x^2-1\right)\left(x^2+1\right)^{-2}\right)$	**2**	**Dep M1** for $\frac{dy}{dx}=k\left(\frac{x^2-1}{x^2+1}\right)^3\times f(x)$ where f(x) is *their* attempt to differentiate **A1** fully correct
	Correct completion to $\frac{16x(x^2-1)^3}{(x^2+1)^5}$	**A1**	
	Alternative		
	Correct derivatives $u=(x^2-1)^4 \quad du=4(x^2-1)^3\times(2x)$ oe and $v=(x^2+1)^4 \quad dv=4(x^2+1)^3\times(2x)$ oe	**(2)**	**M1** for either correct OR for $du=4(x^2-1)^3\times g(x)$ where g(x) is an attempt to differentiate x^2-1 and $dv=4(x^2+1)^3\times h(x)$ where h(x) is an attempt to differentiate x^2+1
	Correct derivative $\frac{dy}{dx}=$ $\frac{(x^2+1)^4\times4(x^2-1)^3(2x)-(x^2-1)^4\times4(x^2+1)^3(2x)}{\left((x^2+1)^4\right)^2}$ or $4(2x)\left(x^2-1\right)^3\left(x^2+1\right)^{-4}-$ $4(2x)\left(x^2-1\right)^4\left(x^2+1\right)^{-5}$	**(2)**	**M1** for an attempt at the product or quotient rule
	Correct completion to $\frac{16x(x^2-1)^3}{(x^2+1)^5}$	**(A1)**	

9(b)(i)	*their* $16x(x^2-1)^3=0$ therefore $x=0,\ x^2-1=0\ \rightarrow x=\pm1$ oe	**B1**	**FT** *their* 16 Allow for *their* $16x(x^2-1)=0$
9(b)(ii)	Any two of x | $-1<x<0$ | 0 | $0<x<1$ $\frac{dy}{dx}$ | positive | 0 | negative x | $0<x<1$ | 1 | $x>1$	**M1**	or equivalent. Must show change in the sign of first derivative around the stationary points **M0** only if second derivative is used

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	x	$0 < x < 1$	1	$x > 1$
	$\frac{dy}{dx}$	negative	0	positive

x	$x < -1$	-1	$-1 < x < 0$
$\frac{dy}{dx}$	negative	0	positive

	$x = 1$ and $x = -1$ are minimum points	**A1**	

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10 **(i)** Write $\ln\left(\frac{2x+1}{2x-1}\right)$ as the difference of two logarithms. [1]

A curve has equation $y = \ln\left(\frac{2x+1}{2x-1}\right) + 4x$ for $x > \frac{1}{2}$.

(ii) Using your answer to part **(i)** show that $\frac{dy}{dx} = \frac{ax^2+b}{4x^2-1}$, where a and b are integers. [4]

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(iii) Hence find the x-coordinate of the stationary point on the curve. [2]

(iv) Determine the nature of this stationary point. [2]

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10(i)	$\ln(2x+1)-\ln(2x-1)$	**B1**	
10(ii)	attempt to differentiate	**M1**	
	$\frac{dy}{dx}=\frac{2}{2x+1}-\frac{2}{2x-1}+4$	**A1**	all correct
	attempt to obtain in required form	**DM1**	
	$=\frac{16x^2-8}{4x^2-1}$	**A1**	A1 all correct
10(iii)	When $\frac{dy}{dx}=0$, $16x^2-8=0$	**M1**	setting $\frac{dy}{dx}=0$ and attempt to solve
	$x=\frac{1}{\sqrt{2}}$ only	**A1**	

10(iv)	$\frac{d^2y}{dx^2}=\frac{32x(4x^2-1)-8x(16x^2-8)}{(4x^2-1)^2}$	**M1**	attempt at second derivative and conclusion or equivalent method
	When $x=\frac{1}{\sqrt{2}}$ $\frac{d^2y}{dx^2}$ is + ve, so minimum	**A1**	

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