Mark scheme

Trigonometric identities and equations

Additional Mathematics 0606 · Questions from Paper 1 and Paper 2

Total marks

72

MARKS FOR EACH QUESTION

1	2	3	4	5	6	7	8	9	10	Total
3	4	5	6	8	9	9	9	9	10	**72**

Question 1 **[3]**

1	$\dfrac{1}{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}}$	M1	M1 for adding fractions in terms of sin/cos/tan/cot correctly
	$=\dfrac{\sin\theta\cos\theta}{\sin^2\theta+\cos^2\theta}$	M1	M1 for use of correct identity
	$=\sin\theta\cos\theta$	A1 **[3]**	A1 for correct solution only
	OR		
	$\dfrac{\tan\theta}{\tan^2\theta+1}$ or $\dfrac{\cot\theta}{\cot^2\theta+1}$	M1	M1 for adding fractions in terms of tan/cot correctly
	$=\dfrac{\tan\theta}{\sec^2\theta}$ or $\dfrac{\cot\theta}{\text{cosec}^2\theta}$	M1	M1 for use of correct identity
	$=\sin\theta\cos\theta$	A1	A1 for correct solution only

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Question 2 **[4]**

2 $\tan^2\theta - \sin^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$		**Marks are awarded only if they can lead to a complete proof for the methods other than those shown below**
$= \dfrac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$	M1	M1 for dealing with tan and a fraction
$= \dfrac{\sin^2\theta\left(1-\cos^2\theta\right)}{\cos^2\theta}$	M1	M1 for factorising
$= \dfrac{\sin^4\theta}{\cos^2\theta}$	M1	M1 for use of identity $\cos^2\theta + \sin^2\theta = 1$
$= \sin^4\theta\sec^2\theta$	A1 [4]	A1 for all correct
Alt solution 1		
Using $\tan^2\theta = \sin^2\theta\sec^2\theta$		
LHS $= \sin^2\theta\sec^2\theta - \sin^2\theta$	M1	M1 use of $\tan^2 x = \sin^2 x\sec^2 x$
$= \sin^2\theta(\sec^2\theta - 1)$	M1	M1 for factorising
$= \sin^2\theta\tan^2\theta$	M1	M1 for use of identity
$= \sin^4\theta\sec^2\theta$	A1	A1 for all correct
Alt solution 2		
RHS $= \sin^4\theta\sec^2\theta$		
$= \dfrac{\sin^2\theta\sin^2\theta}{\cos^2\theta}$	M1	M1 for splitting $\sin^4\theta$ and use of identity
$= \dfrac{\sin^2\theta\left(1-\cos^2\theta\right)}{\cos^2\theta}$	M1	M1 for multiplication
$= \dfrac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$	M1	M1 for writing as two terms and cancelling
$= \dfrac{\sin^2\theta}{\cos^2\theta} - \dfrac{\sin^2\theta\cos^2\theta}{\cos^2\theta}$	A1	A1 for all correct
$= \tan^2\theta - \sin^2\theta$		

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Question 3 [5]

3	$\sin\left(\frac{2x}{3}-\frac{\pi}{3}\right)=\pm\frac{\sqrt{3}}{2}$ or $\tan\left(\frac{2x}{3}-\frac{\pi}{3}\right)=\pm\sqrt{3}$	**B1**	Allow if $\pm$ is missing
	$x=\pi, \frac{3\pi}{2}, \frac{5\pi}{2}, 3\pi$	**4**	**M1dep** on **B1** for obtaining $\frac{2x}{3}-\frac{\pi}{3}=\frac{\pi}{3}$ or any valid value **A1** for one correct solution **A1** for a 2nd correct solution **A1** for a 3rd and 4th correct solutions and no extras in the range

Question 4 [6]

4	$(1+\tan^2 3x)+\tan 3x-3\ [=0]$	**M1**	Uses $\sec^2 3x=1+\tan^2 3x$
	$\tan^2 3x+\tan 3x-2\ [=0]$	**A1**	Writes in solvable form
	(tan $3x$ + 2)(tan $3x$ − 1) [= 0] tan $3x$ = −2, tan $3x$ = 1	**M1**	Factorises or solves their 3-term quadratic in tan $3x$
	Correct triple angle: $3x$ = 116.6 or 296.6 or 45 or 225	**A1**	
	[x =] 15, 75, 38.9 or 38.85[5…] 98.9 or 98.85[5…] and no extras in range $0 \leqslant x \leqslant 120$	**A2**	**A1** for any two correct angles, ignoring extras in range

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Question 5 [8]

5(a)	$\frac{3}{4} = \sin^2 2x$	**B1**	dealing correctly with cosec
	$\sin 2x = \pm\frac{\sqrt{3}}{2}$ $2x = 60,\ 120,\ 240,\ 300$	**M1**	correct method of solution including dealing with $2x$ correctly, may be implied by one correct solution.
	$x = 30,\ 60,\ 120,\ 150$	**A2**	A1 for each correct pair
5(b)	$\tan\left(y - \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$	**M1**	dealing with order of operations to obtain a first solution
	$y - \frac{\pi}{4} = \frac{\pi}{6},\ \frac{7\pi}{6}$	**M1**	M1 for attempt to obtain a second solution
	$y = \frac{5\pi}{12},\ \frac{17\pi}{12}$	**A2**	A1 for each

Question 6 [9]

6 (a)	$2\cos 3x - \frac{\cos 3x}{\sin 3x} = 0$ $\cos 3x\left(2 - \frac{1}{\sin 3x}\right) = 0$	**M1**	for use of $\cot 3x = \frac{\cos 3x}{\sin 3x}$, may be implied
	Leading to $\cos 3x = 0,\ 3x = 90^\circ,\ 270^\circ$	**DM1**	for attempt to solve $\cos 3x = 0$ correctly from correct factorisation to obtain x
	$x = 30^\circ, 90^\circ$	**A1**	**A1** for both, no excess solutions in the range
	and $\sin 3x = \frac{1}{2},\ 3x = 30^\circ,\ 150^\circ$	**DM1**	for attempt to solve $\sin 3x = \frac{1}{2}$ correctly to obtain x
	$x = 10^\circ, 50^\circ$	**A1**	**A1** for both, condone excess solutions
(b)	$\cos\left(y + \frac{\pi}{2}\right) = -\frac{1}{2}$ $y + \frac{\pi}{2} = \frac{2\pi}{3}, \frac{4\pi}{3}$	**M1**	for dealing with sec $\left(y + \frac{\pi}{2}\right)$ correctly
	so $y = \frac{\pi}{6}, \frac{5\pi}{6}$ (0.524, 2.62)	**DM1** **A1, A1**	for correct order of operations, must not mix degrees and radians

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Question 7 **[9]**

7	(a) (i)	$\dfrac{\text{cosec}\,\theta}{\text{cosec}\,\theta-\sin\theta}=\dfrac{\dfrac{1}{\sin\theta}}{\dfrac{1}{\sin\theta}-\sin\theta}$	**M1**	for using $\text{cosec}\,\theta=\dfrac{1}{\sin\theta}$ and either attempt to multiply top and bottom by sin θ or an attempt to combine terms in denominator.
		$=\dfrac{1}{1-\sin^2\theta}$ or $=\dfrac{\dfrac{1}{\sin\theta}}{\dfrac{(1-\sin^2\theta)}{\sin\theta}}$	**DM1**	for correct use of $1\text{-}\sin^2\theta=\cos^2\theta$
		$=\dfrac{1}{\cos^2\theta}$ $=\sec^2\theta$	**A1**	for completing the proof
		Alternative Method using cosec $\dfrac{\text{cosec}\,\theta}{\text{cosec}\,\theta-\sin\theta}=\dfrac{\text{cosec}\,\theta}{\text{cosec}\,\theta-\dfrac{1}{\text{cosec}\,\theta}}$		
		$=\dfrac{\text{cosec}^2\theta}{\text{cosec}^2\theta-1}$	**M1**	for using $\sin\theta=\dfrac{1}{\text{cosec}\,\theta}$ and an attempt to combine terms in denominator.
		$=\dfrac{1+\cot^2\theta}{\cot^2\theta}$	**DM1**	for use of $1+\cot^2\theta=\text{cosec}^2\theta$
		$=\tan^2\theta+1=\sec^2\theta$	**A1**	for completing the proof
	(ii)	$\cos^2\theta=\dfrac{1}{4},\ \cos\theta=\pm\dfrac{1}{2}$ or $\tan^2\theta=3,\ \tan\theta=\pm\sqrt{3}$ or $\sin^2\theta=\dfrac{3}{4},\ \sin\theta=\pm\dfrac{\sqrt{3}}{2}$	**M1**	for using (i) to obtain a value for $\cos^2\theta$, $\tan^2\theta$ or $\sin^2\theta$ and then taking the square root.
		$\theta=60^\circ,120^\circ,240^\circ,300^\circ$	**A1** **A1**	for two correct values for two further correct values and no extras in range.
	(b)	$\tan\left(x+\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt{3}}$ $x=\dfrac{\pi}{6}-\dfrac{\pi}{4},\ \dfrac{7\pi}{6}-\dfrac{\pi}{4},\ \dfrac{13\pi}{6}-\dfrac{\pi}{4}$ $x=\left(-\dfrac{\pi}{12}\right),\ \dfrac{11\pi}{12},\ \dfrac{23\pi}{12}$	**M1** **A1,A1**	for correct order of operations, can be implied by $x=-\dfrac{\pi}{12}$ **A1** for $x=\dfrac{11\pi}{12}$ **A1** for $x=\dfrac{23\pi}{12}$ If there are extra solutions in range in addition to the two correct ones then **A1A0**

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Question 8 **[9]**

8(a)	$\dfrac{\text{cosec}\theta + 1 + \text{cosec}\theta - 1}{\text{cosec}^2\theta - 1}$	**B1**	Allow denominator unsimplified
	$\dfrac{2\text{cosec}\theta}{\cot^2\theta}$	**B1**	
	$\dfrac{2}{\sin\theta} \times \dfrac{\sin^2\theta}{\cos^2\theta}$ $2\sin\theta\sec^2\theta$	**B1**	Sufficient detail must be seen
8(b)	$2\sin 2\phi\sec^2 2\phi = 4\sin 2\phi$ Leading to $\sin 2\phi = 0$ $\phi = \pm 90^\circ,\ 0^\circ$	**2**	**M1** for attempt to solve $\sin 2\phi = 0$ obtaining at least one correct solution **A1** for all solutions
	$2\sin 2\phi\sec^2 2\phi = 4\sin 2\phi$ $\cos 2\phi = (\pm)\dfrac{1}{\sqrt{2}}$	**M1**	For dealing with $\sec^2 2\phi$ to obtain $\cos 2\phi = k$, where $0 \le k \le 1$
	$\phi = \pm 67.5^\circ,\ \pm 22.5^\circ$	**3**	**M1** for solution to obtain at least one correct solution **A1** for a correct pair of solutions **A1** for a second correct pair of solutions with no extra solutions within the range

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Question 9 **[9]**

9(i)	$\dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+1}+\dfrac{\frac{1}{\cos x}+1}{\frac{\sin x}{\cos x}}$	**M1**	Use $\tan x=\dfrac{\sin x}{\cos x}$ and $\sec x=\dfrac{1}{\cos x}$ throughout
	$\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}$	**M1**	**dep** Multiply by cos$x$
	$\dfrac{\sin^2 x+1+2\cos x+\cos^2 x}{(1+\cos x)\sin x}$	**M1**	**dep** Add *their* fractions correctly and expand $(1 + \cos x)^2$ correctly
	$\dfrac{2(1+\cos x)}{(1+\cos x)\sin x}$	**M1**	**dep** Use $\sin^2 x + \cos^2 x = 1$ and take out a factor of 2.
	All correct AG	**A1**	Do not award if brackets missing at any point **or** x missing more than twice **or** x misplaced. Do not credit mixed variables.
	OR $\dfrac{\tan^2 x+(\sec x+1)^2}{\tan x(\sec x+1)}$	**M1**	Add fractions
	$=\dfrac{2\sec^2 x+2\sec x}{\tan x(\sec x+1)}$	**M1**	**dep** Expand brackets correctly and use $1 + \tan^2 x = \sec^2 x$
	$\dfrac{2\sec x}{\tan x}$	**M1**	**dep** Cancel sec$x$ + 1
	$\dfrac{2}{\cos x}\times\dfrac{\cos x}{\sin x}$	**M1**	**dep** Use $\tan x=\dfrac{\sin x}{\cos x}$ and $\sec x=\dfrac{1}{\cos x}$ oe
	All correct AG	**A1**	Do not award if brackets missing at any point **or** x missing more than twice **or** x misplaced. Do not credit mixed variables.
9(ii)	$3\sin^2 x + \sin x - 2 = 0$ oe	**B1**	Obtain three term quadratic.
	$(3\sin x - 2)(\sin x + 1) = 0$	**M1**	Solve three term quadratic
	41.8° awrt	**A1**	
	138.2° awrt	**A1**	Mark final answers This mark is not awarded if there are more solutions in the range.

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Question 10 **[10]**

10(a)	$[\tan 2x = 0 \rightarrow]$ Any two of $2x = 0,\quad 180,\quad 360$	**M1**	
	$[x =]$ 0, 90, 180 nfww and no extras from tan $2x = 0$ in range $0 \leqslant x \leqslant 180$	**A1**	
	$[\tan 2x = 4 \rightarrow]$ $2x =$ 75.96... or 255.96...	**M1**	
	38 [.0] or 37.98[18…] isw 128 [.0] or 127.98[18…] isw and no extras from tan $2x = 4$ in range $0 \leqslant x \leqslant 180$	**A1**	

10(b)	$\sin(y + 1.2) = \frac{1}{4}$	**B1**	
	Two correct values for $y + 1.2$ soi $y + 1.2 = 0.2526[80\ldots]$ $y + 1.2 = \pi - 0.2526[80\ldots]$ or $2.888[91\ldots]$ $y + 1.2 = -\pi - 0.2526[80\ldots]$ or $-3.394[27\ldots]$ soi	**M2**	**M1** for any one correct value for $y + 1.2$
	-0.947 or $-0.9473[19\ldots]$ isw 1.69 or 1.688[91…] isw -4.59 or $-4.594[27\ldots]$ isw and no others in range $-5 < y < 2$	**A3**	**A2** for two correct angles, ignoring extras in range or **A1** for one correct angle, ignoring extras in range

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