Write your name, centre number and candidate number in the boxes below.

Surname	Other names

Centre number | | | | | |

Candidate number | | | | |

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Trigonometric identities and equations

Time allowed: 1 hour 50 minutes

Subject code **0606**

Additional Mathematics

Questions from Paper 1 and Paper 2

You must have: a ruler marked in centimetres and millimetres, a protractor, compasses, a pen, an HB pencil and an eraser.
Questions 1-3, 5-8 (Paper 1): no calculator.
Questions 4, 9-10 (Paper 2): calculator allowed.

Total marks

/ 72

Instructions

- Answer every question in the space provided on this paper.
- Write in black or dark blue ink. Pencil may be used for diagrams and graphs.
- Fill in your name, centre number and candidate number above.
- Show your working: a correct method can earn marks even if the answer is wrong.
- Unless a question says otherwise, give answers that are not exact to 3 significant figures, and angles in degrees to 1 decimal place.

Information

- This paper has 10 questions worth 72 marks in total.
- The marks for each question or part are shown in brackets, for example [3].

Advice

- Read every question through once before you write anything.
- Use the marks in brackets to judge how long to spend on each part.
- If you finish early, go back and check your working.

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Print at 100% (Actual size) so diagrams keep their real measurements.

1 Show that $\dfrac{1}{\tan\theta + \cot\theta} = \sin\theta\cos\theta.$ [3]

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1	$\dfrac{1}{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}}$	M1	M1 for adding fractions in terms of sin/cos/tan/cot correctly
	$=\dfrac{\sin\theta\cos\theta}{\sin^2\theta+\cos^2\theta}$	M1	M1 for use of correct identity
	$=\sin\theta\cos\theta$	A1 **[3]**	A1 for correct solution only
	OR		
	$\dfrac{\tan\theta}{\tan^2\theta+1}$ or $\dfrac{\cot\theta}{\cot^2\theta+1}$	M1	M1 for adding fractions in terms of tan/cot correctly
	$=\dfrac{\tan\theta}{\sec^2\theta}$ or $\dfrac{\cot\theta}{\text{cosec}^2\theta}$	M1	M1 for use of correct identity
	$=\sin\theta\cos\theta$	A1	A1 for correct solution only

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2 Show that $\tan^2\theta - \sin^2\theta = \sin^4\theta\sec^2\theta$. [4]

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2 $\tan^2\theta - \sin^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$		**Marks are awarded only if they can lead to a complete proof for the methods other than those shown below**
$= \dfrac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$	M1	M1 for dealing with tan and a fraction
$= \dfrac{\sin^2\theta\left(1-\cos^2\theta\right)}{\cos^2\theta}$	M1	M1 for factorising
$= \dfrac{\sin^4\theta}{\cos^2\theta}$	M1	M1 for use of identity $\cos^2\theta + \sin^2\theta = 1$
$= \sin^4\theta\sec^2\theta$	A1 [4]	A1 for all correct
Alt solution 1		
Using $\tan^2\theta = \sin^2\theta\sec^2\theta$		
LHS $= \sin^2\theta\sec^2\theta - \sin^2\theta$	M1	M1 use of $\tan^2 x = \sin^2 x\sec^2 x$
$= \sin^2\theta(\sec^2\theta - 1)$	M1	M1 for factorising
$= \sin^2\theta\tan^2\theta$	M1	M1 for use of identity
$= \sin^4\theta\sec^2\theta$	A1	A1 for all correct
Alt solution 2		
RHS $= \sin^4\theta\sec^2\theta$		
$= \dfrac{\sin^2\theta\sin^2\theta}{\cos^2\theta}$	M1	M1 for splitting $\sin^4\theta$ and use of identity
$= \dfrac{\sin^2\theta\left(1-\cos^2\theta\right)}{\cos^2\theta}$	M1	M1 for multiplication
$= \dfrac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$	M1	M1 for writing as two terms and cancelling
$= \dfrac{\sin^2\theta}{\cos^2\theta} - \dfrac{\sin^2\theta\cos^2\theta}{\cos^2\theta}$	A1	A1 for all correct
$= \tan^2\theta - \sin^2\theta$		

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3 Solve the equation $3\,\text{cosec}^2\left(\frac{2x}{3}-\frac{\pi}{3}\right)=4$, for $0<x\leqslant 3\pi$. Give your answers in terms of π. [5]

Trigonometric equations Click for more variations

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3	$\sin\left(\frac{2x}{3}-\frac{\pi}{3}\right)=\pm\frac{\sqrt{3}}{2}$ or $\tan\left(\frac{2x}{3}-\frac{\pi}{3}\right)=\pm\sqrt{3}$	**B1**	Allow if $\pm$ is missing
	$x=\pi, \frac{3\pi}{2}, \frac{5\pi}{2}, 3\pi$	**4**	**M1dep** on **B1** for obtaining $\frac{2x}{3}-\frac{\pi}{3}=\frac{\pi}{3}$ or any valid value **A1** for one correct solution **A1** for a 2nd correct solution **A1** for a 3rd and 4th correct solutions and no extras in the range

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4 Solve the equation $\sec^2 3x + \tan 3x - 3 = 0$ for $0° \leqslant x \leqslant 120°$. [6]

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4	$(1 + \tan^2 3x) + \tan 3x - 3 \ [= 0]$	**M1**	Uses $\sec^2 3x = 1 + \tan^2 3x$
	$\tan^2 3x + \tan 3x - 2 \ [= 0]$	**A1**	Writes in solvable form
	$(\tan 3x + 2)(\tan 3x - 1) \ [= 0]$ $\tan 3x = -2, \ \tan 3x = 1$	**M1**	Factorises or solves their 3-term quadratic in $\tan 3x$
	Correct triple angle: $3x = 116.6$ or 296.6 or 45 or 225	**A1**	
	$[x =]$ 15, 75, 38.9 or 38.85[5…] 98.9 or 98.85[5…] and no extras in range $0 \leqslant x \leqslant 120$	**A2**	**A1** for any two correct angles, ignoring extras in range

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5 **(a)** Solve $3\operatorname{cosec} 2x - 4\sin 2x = 0$ for $0° \leqslant x \leqslant 180°$. [4]

(b) Solve $3\tan\left(y - \frac{\pi}{4}\right) = \sqrt{3}$ for $0 \leqslant y \leqslant 2\pi$ radians, giving your answers in terms of π. [4]

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5(a)	$\frac{3}{4} = \sin^2 2x$	**B1**	dealing correctly with cosec
	$\sin 2x = \pm\frac{\sqrt{3}}{2}$ $2x = 60,\ 120,\ 240,\ 300$	**M1**	correct method of solution including dealing with $2x$ correctly, may be implied by one correct solution.
	$x = 30,\ 60,\ 120,\ 150$	**A2**	A1 for each correct pair
5(b)	$\tan\left(y - \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$	**M1**	dealing with order of operations to obtain a first solution
	$y - \frac{\pi}{4} = \frac{\pi}{6},\ \frac{7\pi}{6}$	**M1**	M1 for attempt to obtain a second solution
	$y = \frac{5\pi}{12},\ \frac{17\pi}{12}$	**A2**	A1 for each

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6 **(a)** Solve $2\cos 3x = \cot 3x$ for $0° \leqslant x \leqslant 90°$. [5]

(b) Solve $\sec\left(y + \frac{\pi}{2}\right) = -2$ for $0 \leqslant y \leqslant \pi$ radians. [4]

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6 (a)	$2\cos 3x - \dfrac{\cos 3x}{\sin 3x} = 0$ $\cos 3x\left(2 - \dfrac{1}{\sin 3x}\right) = 0$	**M1**	for use of $\cot 3x = \dfrac{\cos 3x}{\sin 3x}$, may be implied
	Leading to $\cos 3x = 0,\ 3x = 90^\circ,\ 270^\circ$	**DM1**	for attempt to solve $\cos 3x = 0$ correctly from correct factorisation to obtain x
	$x = 30^\circ, 90^\circ$	**A1**	**A1** for both, no excess solutions in the range
	and $\sin 3x = \dfrac{1}{2},\ 3x = 30^\circ,\ 150^\circ$	**DM1**	for attempt to solve $\sin 3x = \dfrac{1}{2}$ correctly to obtain x
	$x = 10^\circ, 50^\circ$	**A1**	**A1** for both, condone excess solutions
(b)	$\cos\left(y + \dfrac{\pi}{2}\right) = -\dfrac{1}{2}$ $y + \dfrac{\pi}{2} = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$	**M1**	for dealing with sec $\left(y + \dfrac{\pi}{2}\right)$ correctly
		DM1	for correct order of operations, must not mix degrees and radians
	so $y = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ (0.524, 2.62)	**A1, A1**	

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7 **(a)** **(i)** Show that $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} = \sec^2\theta$. [3]

(ii) Hence solve $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} = 4$ for $0° < \theta < 360°$. [3]

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(b) Solve $\sqrt{3}\tan\left(x+\frac{\pi}{4}\right)=1$ for $0<x<2\pi$, giving your answers in terms of π. [3]

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Trigonometric identities and proofs Click for more variations

7	(a) (i)	$\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-\sin\theta}=\dfrac{\dfrac{1}{\sin\theta}}{\dfrac{1}{\sin\theta}-\sin\theta}$	**M1**	for using $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$ and either attempt to multiply top and bottom by sin θ or an attempt to combine terms in denominator.
		$=\dfrac{1}{1-\sin^2\theta}$ or $=\dfrac{\dfrac{1}{\sin\theta}}{\dfrac{(1-\sin^2\theta)}{\sin\theta}}$	**DM1**	for correct use of $1-\sin^2\theta=\cos^2\theta$
		$=\dfrac{1}{\cos^2\theta}$ $=\sec^2\theta$	**A1**	for completing the proof
		Alternative Method using cosec		
		$\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-\sin\theta}=\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-\dfrac{1}{\operatorname{cosec}\theta}}$		
		$=\dfrac{\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta-1}$	**M1**	for using $\sin\theta=\dfrac{1}{\operatorname{cosec}\theta}$ and an attempt to combine terms in denominator.
		$=\dfrac{1+\cot^2\theta}{\cot^2\theta}$	**DM1**	for use of $1+\cot^2\theta=\operatorname{cosec}^2\theta$
		$=\tan^2\theta+1=\sec^2\theta$	**A1**	for completing the proof
	(ii)	$\cos^2\theta=\dfrac{1}{4},\ \cos\theta=\pm\dfrac{1}{2}$ or $\tan^2\theta=3,\ \tan\theta=\pm\sqrt{3}$ or $\sin^2\theta=\dfrac{3}{4},\ \sin\theta=\pm\dfrac{\sqrt{3}}{2}$	**M1**	for using (i) to obtain a value for $\cos^2\theta$, $\tan^2\theta$ or $\sin^2\theta$ and then taking the square root.
		$\theta=60^\circ,120^\circ,240^\circ,300^\circ$	**A1** **A1**	for two correct values for two further correct values and no extras in range.
	(b)	$\tan\left(x+\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt{3}}$	**M1**	for correct order of operations, can be implied by $x=-\dfrac{\pi}{12}$
		$x=\dfrac{\pi}{6}-\dfrac{\pi}{4},\ \dfrac{7\pi}{6}-\dfrac{\pi}{4},\ \dfrac{13\pi}{6}-\dfrac{\pi}{4}$ $x=\left(-\dfrac{\pi}{12}\right),\ \dfrac{11\pi}{12},\ \dfrac{23\pi}{12}$	**A1,A1**	**A1** for $x=\dfrac{11\pi}{12}$ **A1** for $x=\dfrac{23\pi}{12}$ If there are extra solutions in range in addition to the two correct ones then **A1A0**

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8 **(a)** Show that $\dfrac{1}{\operatorname{cosec}\theta - 1} + \dfrac{1}{\operatorname{cosec}\theta + 1} = 2\sin\theta\sec^2\theta$. [3]

(b) Hence solve the equation $\dfrac{1}{\operatorname{cosec} 2\phi - 1} + \dfrac{1}{\operatorname{cosec} 2\phi + 1} = 4\sin 2\phi$, for $-90° \leqslant \phi \leqslant 90°$. [6]

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8(a)	$\dfrac{\text{cosec}\,\theta + 1 + \text{cosec}\,\theta - 1}{\text{cosec}^2\theta - 1}$	**B1**	Allow denominator unsimplified
	$\dfrac{2\text{cosec}\,\theta}{\cot^2\theta}$	**B1**	
	$\dfrac{2}{\sin\theta} \times \dfrac{\sin^2\theta}{\cos^2\theta}$ $2\sin\theta\sec^2\theta$	**B1**	Sufficient detail must be seen
8(b)	$2\sin 2\phi\sec^2 2\phi = 4\sin 2\phi$ Leading to $\sin 2\phi = 0$ $\phi = \pm 90^\circ,\ 0^\circ$	**2**	**M1** for attempt to solve $\sin 2\phi = 0$ obtaining at least one correct solution **A1** for all solutions
	$2\sin 2\phi\sec^2 2\phi = 4\sin 2\phi$ $\cos 2\phi = (\pm)\dfrac{1}{\sqrt{2}}$	**M1**	For dealing with $\sec^2 2\phi$ to obtain $\cos 2\phi = k$, where $0 \le k \le 1$
	$\phi = \pm 67.5^\circ,\ \pm 22.5^\circ$	**3**	**M1** for solution to obtain at least one correct solution **A1** for a correct pair of solutions **A1** for a second correct pair of solutions with no extra solutions within the range

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9 **(i)** Show that $\dfrac{\tan x}{1+\sec x}+\dfrac{1+\sec x}{\tan x}=\dfrac{2}{\sin x}$. [5]

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(ii) Hence solve the equation $\dfrac{\tan x}{1+\sec x}+\dfrac{1+\sec x}{\tan x}=1+3\sin x$ for $0^\circ \leqslant x \leqslant 180^\circ$. [4]

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9(i)	$\dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+1}+\dfrac{\frac{1}{\cos x}+1}{\frac{\sin x}{\cos x}}$	**M1**	Use $\tan x=\dfrac{\sin x}{\cos x}$ and $\sec x=\dfrac{1}{\cos x}$ throughout
	$\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}$	**M1**	**dep** Multiply by cos$x$
	$\dfrac{\sin^2 x+1+2\cos x+\cos^2 x}{(1+\cos x)\sin x}$	**M1**	**dep** Add *their* fractions correctly and expand $(1 + \cos x)^2$ correctly
	$\dfrac{2(1+\cos x)}{(1+\cos x)\sin x}$	**M1**	**dep** Use $\sin^2 x + \cos^2 x = 1$ and take out a factor of 2.
	All correct AG	**A1**	Do not award if brackets missing at any point **or** x missing more than twice **or** x misplaced. Do not credit mixed variables.
	OR $\dfrac{\tan^2 x+(\sec x+1)^2}{\tan x(\sec x+1)}$	**M1**	Add fractions
	$=\dfrac{2\sec^2 x+2\sec x}{\tan x(\sec x+1)}$	**M1**	**dep** Expand brackets correctly and use $1 + \tan^2 x = \sec^2 x$
	$\dfrac{2\sec x}{\tan x}$	**M1**	**dep** Cancel sec$x$ + 1
	$\dfrac{2}{\cos x}\times\dfrac{\cos x}{\sin x}$	**M1**	**dep** Use $\tan x=\dfrac{\sin x}{\cos x}$ and $\sec x=\dfrac{1}{\cos x}$ oe
	All correct AG	**A1**	Do not award if brackets missing at any point **or** x missing more than twice **or** x misplaced. Do not credit mixed variables.
9(ii)	$3\sin^2 x + \sin x - 2 = 0$ oe	**B1**	Obtain three term quadratic.
	$(3\sin x - 2)(\sin x + 1) = 0$	**M1**	Solve three term quadratic
	41.8° awrt	**A1**	
	138.2° awrt	**A1**	Mark final answers This mark is not awarded if there are more solutions in the range.

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10 **(a)** Solve the equation $\tan^2 2x - 4\tan 2x = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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(b) Solve the equation $\operatorname{cosec}(y+1.2)=4$, where y is in radians and $-5<y<2$. [6]

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10(a)	$[\tan 2x = 0 \rightarrow]$ Any two of $2x = 0,\quad 180,\quad 360$	**M1**	
	$[x =]$ 0, 90, 180 nfww and no extras from tan $2x = 0$ in range $0 \leqslant x \leqslant 180$	**A1**	
	$[\tan 2x = 4 \rightarrow]$ $2x =$ 75.96... or 255.96...	**M1**	
	38 [.0] or 37.98[18…] isw 128 [.0] or 127.98[18…] isw and no extras from tan $2x = 4$ in range $0 \leqslant x \leqslant 180$	**A1**	

10(b)	$\sin(y + 1.2) = \frac{1}{4}$	**B1**	
	Two correct values for $y + 1.2$ soi $y + 1.2 = 0.2526[80\ldots]$ $y + 1.2 = \pi - 0.2526[80\ldots]$ or $2.888[91\ldots]$ $y + 1.2 = -\pi - 0.2526[80\ldots]$ or $-3.394[27\ldots]$ soi	**M2**	**M1** for any one correct value for $y + 1.2$
	-0.947 or $-0.9473[19\ldots]$ isw 1.69 or 1.688[91…] isw -4.59 or $-4.594[27\ldots]$ isw and no others in range $-5 < y < 2$	**A3**	**A2** for two correct angles, ignoring extras in range or **A1** for one correct angle, ignoring extras in range

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