Mark scheme

Vectors

Additional Mathematics 0606 · Questions from Paper 1 and Paper 2

Total marks

71

MARKS FOR EACH QUESTION

1	2	3	4	5	6	7	8	9	10	Total
5	6	6	6	6	7	8	8	9	10	**71**

Question 1 **[5]**

1(a)	$\pm\begin{pmatrix}-5\\12\end{pmatrix}$	**B1**	
	$\begin{pmatrix}-5\\12\end{pmatrix}$	**B1**	
1(b)	13	**B1**	**FT** on *their* (a)
1(c)	$3\left(\text{their } \overrightarrow{AB}\right) = 2\overrightarrow{OX} - 2\begin{pmatrix}2\\-6\end{pmatrix}$	**M1**	Condone $3\left(\text{their } \overrightarrow{AB}\right) = 2\begin{pmatrix}2\\-6\end{pmatrix} - 2OX$
	$\begin{pmatrix}-\frac{11}{2}\\12\end{pmatrix}$	**A1**	

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Question 2 **[6]**

2(a)	$\left\|\begin{pmatrix}-12\\5\end{pmatrix}\right\| = 13$	**B1**	For magnitude, may be implied by a correct **v**
	$\mathbf{v} = \begin{pmatrix}-36\\15\end{pmatrix}$ or $3\begin{pmatrix}-12\\5\end{pmatrix}$	**B1**	Must be a vector
2(a) Alternative	If $t\left\|\begin{pmatrix}-12\\5\end{pmatrix}\right\| = 39$, $t = 3$	**B1**	For value of t, may be implied by a correct **v**
	$\mathbf{v} = \begin{pmatrix}-36\\15\end{pmatrix}$ or $3\begin{pmatrix}-12\\5\end{pmatrix}$	**B1**	
2(b)		**M1**	For equating like vectors at least once
	$17r + 2s + 3 = 0$ $2r + 6s + 9 = 0$	**M1**	**Dep** For solution of resulting equations to obtain 2 solutions
	$r = 0$	**A1**	
	$s = -\frac{3}{2}$ oe	**A1**	

Question 3 **[6]**

3(i)	$\|their(\mathbf{a} + \mathbf{c})\| = \sqrt{their(5^2 + 14^2)}$	**M1**	
	$\sqrt{221}$	**A1**	mark final answer
3(ii)	$[(2 + m)\mathbf{i} + (3 - 5m)\mathbf{j}$ therefore] *their* $(2 + m) = 0$	**M1**	for attempting to form $\mathbf{a} + m\mathbf{b}$ and equate the scalar of the **i** component to 0
	$m = -2$ only	**A1**	implies **M1**
3(iii)	$[(2n - 1)\mathbf{i} + (3n + 5)\mathbf{j} = 3\mathbf{i} + 11\mathbf{j}$ or $n(2\mathbf{i} + 3\mathbf{j}) = (3\mathbf{i} + 11\mathbf{j}) + (\mathbf{i} - 5\mathbf{j})$ oe leading to] $2n - 1 = 3$ or $3n + 5 = 11$ oe, soi	**M1**	
	$n = 2$ only	**A1**	implies **M1**

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Question 4 **[6]**

4(i)	Uses correct triangle with v_w opposite 10° Sides of 300 and 280 include 10°	**M1**	
	Use cosine rule	**M1**	$v_w^{\,2} = 300^2 + 280^2 - 2\times 300\times 280\cos 10$
	$v_w = 54.3$	**A1**	
4(ii)	Use sine rule	**M1**	$\frac{280}{\sin\alpha} = \frac{54.3}{\sin 10°}$
	$\alpha = 63°$ or $64°$	**A1**	
	Bearing 117° or 116°	**A1**	

Question 5 **[6]**

5	**(i)**	$\overrightarrow{OX} = \mu(a+b)$	**B1**	
	(ii)	$\overrightarrow{RP} = b-3a$ or $\overrightarrow{RX} = \lambda(b-3a)$ oe	**B1**	
		$\overrightarrow{OX} = 3a + \lambda(b-3a)$	**B1**	
	(iii)	$\overrightarrow{OX} = \overrightarrow{OX}$ and equate both coefficients $\mu = 3-3\lambda$ $\mu = \lambda$	**M1**	
		$\mu = \lambda = 0.75$	**A1**	
		$\frac{RX}{XP} = 3$ or 3:1	**A1ft**	$\frac{\lambda}{1-\lambda}$

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Question 6 **[7]**

6(i)	$v = 20 \times \frac{1}{\sqrt{3^2 + 4^2}} \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ oe	**M1**	finding and using the magnitude of $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$
	$v = \begin{pmatrix} 12 \\ 16 \end{pmatrix}$	**A1**	
6(ii)	$\boldsymbol{r}_p = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 12 \\ 16 \end{pmatrix} t$	**M1**	correct use of position vector and *their* velocity vector
		A1	
6(iii)	$\begin{pmatrix} 17 \\ 18 \end{pmatrix} + \begin{pmatrix} 8 \\ 12 \end{pmatrix} t = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 12 \\ 16 \end{pmatrix} t$ Leading to $17 + 8t = 1 + 12t$ or $18 + 12t = 2 + 16t$	**M1**	equating position vectors of both particles at time t and solve either equation for t
	$t = 4$	**A1**	
	Position vector of collision $\begin{pmatrix} 49 \\ 66 \end{pmatrix}$	**A1**	

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Question 7 **[8]**

7(a)	$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$	**B1**	
	$\sqrt{4^2 + 8^2}$	**M1**	**FT** *their* $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$
	$\frac{1}{\sqrt{80}}\begin{pmatrix} 4 \\ 8 \end{pmatrix}$ oe isw	**A1**	**FT** provided working shown
7(b)	$\begin{pmatrix} 6 \\ -5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 10+x \\ 3+y \end{pmatrix}$ oe	**M1**	
	$x = 2,\ y = -13$	**A1**	
7(c)	$\overrightarrow{OE} = \frac{1}{1+\lambda}\begin{pmatrix} 12 \\ 7 \end{pmatrix}$ oe seen	**B1**	
	Solves *their* $\frac{7}{1+\lambda} = 3$	**M1**	
	$\lambda = \frac{4}{3}$ oe	**A1**	
	Alternative $\overrightarrow{OE} = \begin{pmatrix} x \\ 3 \end{pmatrix}$ $\frac{12}{7} = \frac{x}{3}$ $x = \frac{36}{7}$	**(B1)**	
	$\frac{1+\lambda}{1} = \frac{12}{36/7}$	**(M1)**	**FT** *their x*
	$\lambda = \frac{4}{3}$	**(A1)**	

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Question 8 **[8]**

8	**(a)**	$\overrightarrow{AB} = 15\mathbf{b} - 5\mathbf{a} = 5(3\mathbf{b} - \mathbf{a})$ or $\overrightarrow{BC} = 24\mathbf{b} - 3\mathbf{a} - 15\mathbf{b} = 3(3\mathbf{b} - \mathbf{a})$ or $\overrightarrow{AC} = 24\mathbf{b} - 3\mathbf{a} - 5\mathbf{a} = 8(3\mathbf{b} - \mathbf{a})$	B1 B1	Any correct simplified vector Any second simplified vector
		Comment: e.g. the vectors are scalar multiples of each other AND they have a common point (A, B or C as appropriate)	B1dep	Dep on both B marks being awarded.
	(b) (i)	$2\mathbf{i} + 11\mathbf{j}$ soi $\Rightarrow \sqrt{2^2 + 11^2}$	B1	
		$\sqrt{125}$ or $5\sqrt{5}$ or 11.2 (3 s.f.) or better)	B1fT	ft *their* $2\mathbf{i} + 11\mathbf{j}$ (not $\overrightarrow{OP}$ or $\overrightarrow{OQ}$)
	(ii)	$\frac{1}{5\sqrt{5}}(2\mathbf{i} + 11\mathbf{j})$ isw	B1fT	ft *their* answers from **(i)**
	(iii)	$\frac{\mathbf{i} - 4\mathbf{j} + 3\mathbf{i} + 7\mathbf{j}}{2}$ or $\mathbf{i} - 4\mathbf{j} + \frac{2\mathbf{i} + 11\mathbf{j}}{2}$ or $3\mathbf{i} + 7\mathbf{j} - \frac{2\mathbf{i} + 11\mathbf{j}}{2}$	M1	
		$2\mathbf{i} + 1.5\mathbf{j}$	A1	

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Question 9 **[9]**

9(a)	$\overrightarrow{OP}=\frac{4}{7}\mathbf{b}+\lambda\left(\mathbf{c}-\frac{4}{7}\mathbf{b}\right)$ and $\overrightarrow{OP}=\mathbf{b}+\mu\left(\frac{4}{7}\mathbf{c}-\mathbf{b}\right)$	**B3**	**B2** for $\overrightarrow{OP}=\frac{4}{7}\mathbf{b}+\lambda\left(\mathbf{c}-\frac{4}{7}\mathbf{b}\right)$ or $\overrightarrow{OP}=\mathbf{b}+\mu\left(\frac{4}{7}\mathbf{c}-\mathbf{b}\right)$ oe or **B1** for $\overrightarrow{OP}=\left(their\frac{4}{7}\right)\mathbf{b}+\lambda\left(\mathbf{c}-\left(their\frac{4}{7}\right)\mathbf{b}\right)$ or $\overrightarrow{OP}=\mathbf{b}+\mu\left(\left(their\frac{4}{7}\right)\mathbf{c}-\mathbf{b}\right)$ or
	Equates components e.g.: $\left(their\frac{4}{7}\right)(1-\lambda)=(1-\mu)$ or $\lambda=their\frac{4}{7}\mu$	**M1**	**FT** providing at least **B1** awarded and two expressions for $\overrightarrow{OP}$ in terms of **b**, **c**, λ and μ found
	$\frac{4}{7}(1-\lambda)=(1-\mu)\quad\lambda=\frac{4}{7}\mu$ oe	**A1**	
	$\lambda=\frac{4}{11}$ and $\mu=\frac{7}{11}$ oe and conclusion $AP : AC = 4 : 11$ therefore $AP : PC = 4 : 7$ $BP : BD = 7 : 11$ therefore $DP : PB = 4 : 7$ oe	**A2**	**A1** for $\lambda=\frac{4}{11}$ or $\mu=\frac{7}{11}$ oe

9(b)	$\overrightarrow{OP}=\frac{4}{11}(\mathbf{b}+\mathbf{c})$ or $\overrightarrow{OP}=\frac{4}{11}\mathbf{b}+\frac{4}{11}\mathbf{c}$ and $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are scalar multiples of each other and have a point in common oe	**2**	**M1** for $\overrightarrow{OP}=\frac{4}{11}(\mathbf{b}+\mathbf{c})$ or $\overrightarrow{OP}=\frac{4}{11}\mathbf{b}+\frac{4}{11}\mathbf{c}$

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Question 10 **[10]**

10(a)	Velocity vector = $\begin{pmatrix}-10\\10.5\end{pmatrix}$ or $\frac{1}{2}\begin{pmatrix}-20\\21\end{pmatrix}$	**B2**	**B1** for $\left\lvert\begin{pmatrix}-20\\21\end{pmatrix}\right\rvert = 29$ soi or one correct element
10(b)	$\begin{pmatrix}3\\5\end{pmatrix}+\begin{pmatrix}-10\\10.5\end{pmatrix}t$ oe	**2**	**M1 FT** on *their* velocity vector from part (a) if not correct.
10(c)	$\overrightarrow{PQ} = \begin{pmatrix}5t-4\\-3t-2\end{pmatrix}$ oe	**2**	**Dep M1** must have **M1** in part (b) for $\left(\begin{pmatrix}-1\\3\end{pmatrix}+\begin{pmatrix}-5\\7.5\end{pmatrix}t\right) - $ *their* answer to part (b) or *their* answer to part b $-\left(\begin{pmatrix}-1\\3\end{pmatrix}+\begin{pmatrix}-5\\7.5\end{pmatrix}t\right)$
	$PQ = \sqrt{(5t-4)^2+(-3t-2)^2}$	**M1**	**Dep** on previous **M** mark for using *their* $\overrightarrow{PQ}$
	$PQ = \sqrt{34t^2-28t+20}$	**A1**	
10(d)	For collision $34t^2-28t+20=0$ soi	**M1**	Need to be using *their* quadratic PQ^2 from part (c) or forming a quadratic from their $\overrightarrow{PQ}$ from part (c)
	Discriminant = -1936 So no real roots so no collision oe	**A1**	Discriminant must be correct, but allow unsimplified

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